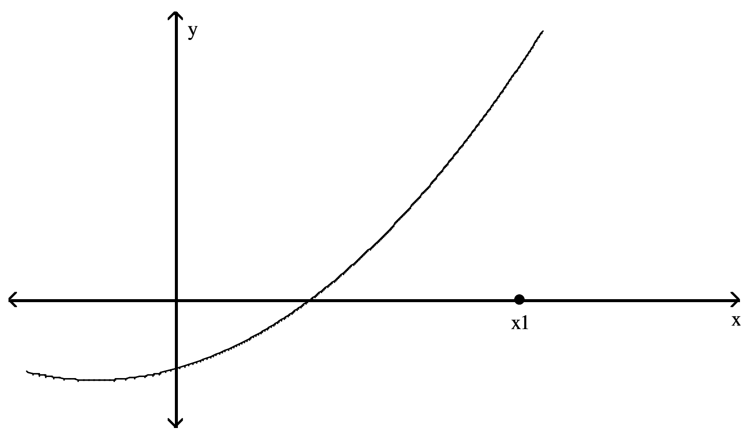
Name________________________________

Newton's Method Activity

Part 1: The Geometric Explanation

To approximate the x–intercept of a function, Newton's Method uses the tangent line to the function at a value that is close to the correct x–intercept. The root of the linear equation is mathematically very easy to find, whereas the exact root of the orignal function may be extremely difficult to find. The process begins by guessing an initial value for the intercept that is close to the actual intercept.

Given the initial guess x_1 shown below, use a straight edge to draw the tangent line to the function that passes through the point $(x_1, f(x_1))$.



Label the x–intercept of your tangent line x_2. This point is a better approximation to the actual root of the function.

Repeat the process using a straight edge to draw the tangent line to the function that passes through the point $(x_2, f(x_2))$. Label the x–intercept of this line x_3. Notice that this newest approximation x_3 is an even better approximation of the actual root of the function.

Notice that in this particular example, our first guess of x_1 will lead to successive guesses that converge to the actual root of the function. If you continued this process, the intercept of each successive line would get closer and closer to the actual intercept of the function.

Can you find a different point x_1 in this figure that would cause Newton's Method to fail? If so, mark it on the graph and explain why the method fails.

Part 2: Developing the formula for approximation x_{n+1}

Using the point–slope equation of a line, find the tangent line to the graph of $f(x)$ at the point $(x_1, f(x_1))$ using derivative notation for the slope.

The second approximation x_2 is the x–intercept of this line: the point $(x_2, 0)$. Substitute $(x_2, 0)$ for (x, y) in the equation, and solve for x_2.

$x_2 =$ ______________________

The equation of the tangent line to the graph of $f(x)$ at the new point $(x_2, f(x_2))$ is:

The third approximation x_3 is the x–intercept of this line: the point $(x_3, 0)$. Substitute $(x_3, 0)$ for (x, y) in the equation, and solve for x_3.

$x_3 =$ ______________________

This pattern would repeat over and over. Looking at your results above, write down the formula for x_{n+1} given that you have the equation of the line through $(x_n, f(x_n))$.

$x_{n+1} =$ ______________________

Part 3: Estimating a root

Use Newton's method to estimate the root of the function. Start with the given value of x_1. You will need a calculator!

$f(x) = x^3 + x - 1$

Write your <u>specific</u> formula for x_{n+1}

$x_1 = 1$

$x_2 =$ ________________

$x_3 =$ ________________