

Activity - Secant Lines and Tangent Lines

Part 1. Suppose that f is the function given by the graph below and that a and $a + h$ are the input values as labeled on the x -axis. Use the graph in Figure below to answer the following questions.

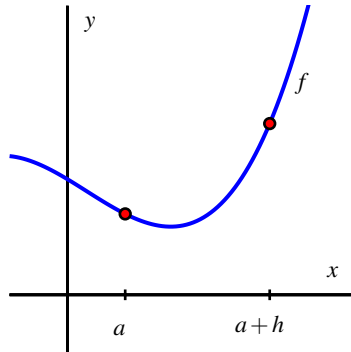


Figure: Plot of $y = f(x)$

- (a) Locate and label the points $(a, f(a))$ and $(a + h, f(a + h))$ on the graph.
- (b) Construct a right triangle whose hypotenuse is the line segment from $(a, f(a))$ to $(a + h, f(a + h))$. What are the lengths of the respective legs of this triangle?
- (c) What is the slope of the line that connects the points $(a, f(a))$ and $(a + h, f(a + h))$?
- (d) Write a meaningful sentence that explains how the average rate of change of the function on a given interval and the slope of a related line are connected.

Part 2. Consider the graph of $y = f(x)$ provided in Figure below.

(a) On the graph of $y = f(x)$, sketch and label the following quantities:

- the secant line to $y = f(x)$ on the interval $[-3, -1]$ and the secant line to $y = f(x)$ on the interval $[0, 2]$.
- the tangent line to $y = f(x)$ at $x = -3$ and the tangent line to $y = f(x)$ at $x = 0$.

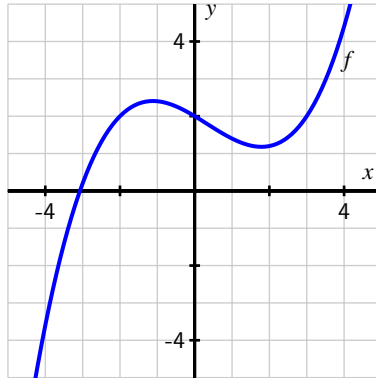


Figure: Plot of $y = f(x)$.

(b) What is the approximate value of the average rate of change of f on $[-3, -1]$? On $[0, 2]$? How are these values related to your work in (a)?

(c) What is the approximate value of the instantaneous rate of change of f at $x = -3$? At $x = 0$? How are these values related to your work in (a)?

Part 3. For each of the following prompts, sketch a graph of a continuous function that has the stated properties.

(a) $y = f(x)$ such that

- the average rate of change of f on $[-3, 0]$ is -2 and the average rate of change of f on $[1, 3]$ is 0.5 , and
- the instantaneous rate of change of f at $x = -1$ is -1 and the instantaneous rate of change of f at $x = 2$ is 1 .

(b) $y = g(x)$ such that

- $\frac{g(3)-g(-2)}{5} = 0$ and $\frac{g(1)-g(-1)}{2} = -1$, and
- $g'(2) = 1$ and $g'(-1) = 0$