Name______________________________

Activity - Areas, Distances and Velocities

Part 1. Suppose that a person is taking a walk along a long straight path and walks at a constant rate of 3 miles per hour.

(a) On the left-hand axes provided in Figure, sketch a labeled graph of the velocity function $v(t) = 3$. Note that while the scale on the two sets of axes is the same,

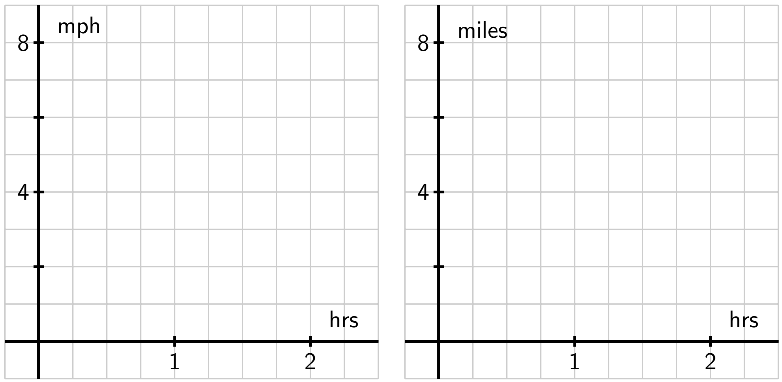

Figure: At left, axes for plotting $y = v(t)$; at right, for plotting $y = s(t)$.

the units on the right-hand axes differ from those on the left. The right-hand axes will be used in question (d).

(b) How far did the person travel during the two hours? How is this distance related to the area of a certain region under the graph of $y = v(t)$?

(c) Find an algebraic formula, $s(t)$, for the position of the person at time t, assuming that $s(0) = 0$. Explain your thinking.

(d) On the right-hand axes provided in the figure, sketch a labeled graph of the position function $y = s(t)$.

(e) For what values of t is the position function s increasing? Explain why this is the case using relevant information about the velocity function v.

Part 2.

A ball is tossed vertically in such a way that its velocity function is given by $v(t) = 32 - 32t$, where t is measured in seconds and v in feet per second. Assume that this function is valid for $0 \le t \le 2$.

(a) For what values of t is the velocity of the ball positive? What does this tell you about the motion of the ball on this interval of time values?

(b) Find an antiderivative, s, of v that satisfies $s(0) = 0$.

(c) Compute the value of $s(1) - s(\frac{1}{2})$. What is the meaning of the value you find?

(d) Using the graph of $y = v(t)$ provided in the figure, find the exact area of the region under the velocity curve between $t = \frac{1}{2}$ and $t = 1$. What is the meaning of the value you find?

(e) Answer the same questions as in (c) and (d) but instead using the interval $[0,1]$.

(f) What is the value of $s(2) - s(0)$? What does this result tell you about the flight of the ball? How is this value connected to the provided graph of $y = v(t)$? Explain.

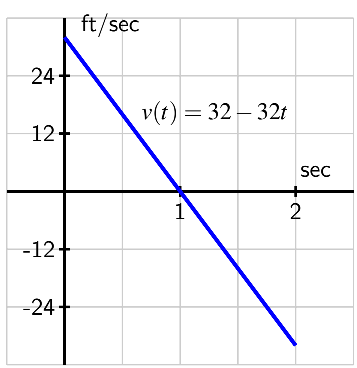

Figure: The graph of $y = v(t)$.

Part 3. A person walking along a straight path has her velocity in miles per hour at time t given by the function $v(t) = 0.25t^3 - 1.5t^2 + 3t + 0.25$, for times in the interval $0 \le t \le 2$. The graph of this function is also given in each of the three diagrams in Figure. Note that in each diagram, we use four rectangles to estimate the area under

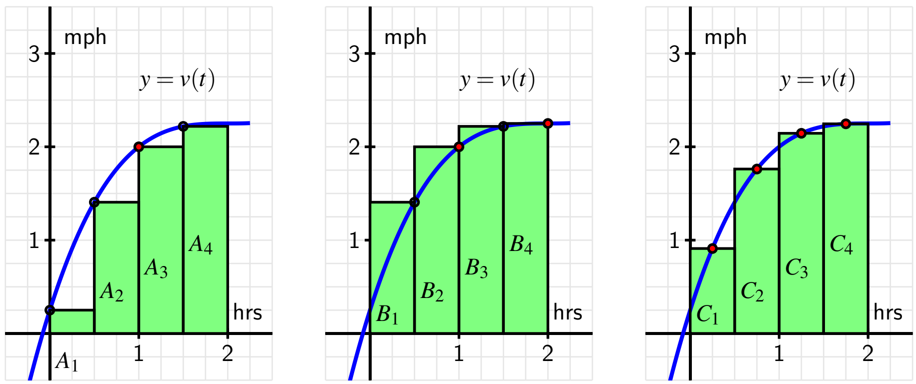

Figure: Three approaches to estimating the area under $y = v(t)$ on the interval [0,2].

$y = v(t)$ on the interval $[0,2]$, but the method by which the four rectangles' respective heights are decided varies among the three individual graphs.

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(a) How are the heights of rectangles in the left-most diagram being chosen? Explain, and hence determine the value of

$$S = A_1 + A_2 + A_3 + A_4$$

by evaluating the function $y = v(t)$ at appropriately chosen values and observing the width of each rectangle. Note, for example, that

$$A_3 = v(1) \cdot \frac{1}{2} = 2 \cdot \frac{1}{2} = 1.$$

(b) Explain how the heights of rectangles are being chosen in the middle diagram and find the value of

$$T = B_1 + B_2 + B_3 + B_4.$$

(c) Likewise, determine the pattern of how heights of rectangles are chosen in the right-most diagram and determine

$$U = C_1 + C_2 + C_3 + C_4.$$

(d) Of the estimates S, T, and U, which do you think is the best approximation of D, the total distance the person traveled on $[0,2]$? Why?

Part 4.

Suppose that an object moving along a straight line path has its velocity in feet per second at time t in seconds given by $v(t) = \frac{2}{9}(t-3)^2 + 2$.

(a) Carefully sketch the region whose exact area will tell you the value of the distance the object traveled on the time interval $1 \le t \le 5$.

(b) Estimate the distance traveled on $[1, 5]$ by computing L_4, R_4, and M_4.

(c) Does averaging L_4 and R_4 result in the same value as M_4? If not, what do you think the average of L_4 and R_4 measures?

(d) For this question, think about an arbitrary function f, rather than the particular function v given above. If f is positive and increasing on $[a, b]$, will L_n over-estimate or under-estimate the exact area under f on $[a, b]$? Will R_n over- or under-estimate the exact area under f on $[a, b]$? Explain.

At http://gvsu.edu/s/a9, the applet noted earlier, by unchecking the "relative" box at the top left, and instead checking "random," we can easily explore the effect of using random point locations in subintervals on a given Riemann sum. In computational practice, we most often use L_n, R_n, or M_n, while the random Riemann sum is useful in theoretical discussions. In the following activity, we investigate several different Riemann sums for a particular velocity function.