Name____

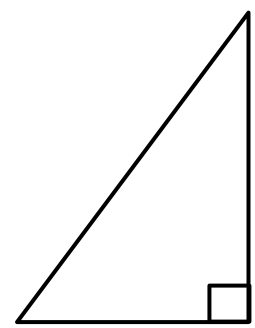
Activity – Law of Sines

To "solve" a triangle means to find the measure of all of its angles and the lengths of all of its sides.

Part 1. Right Triangles

When we have a right triangle, we call the legs *a* and *b* and the hypotenuse *c*. The angles opposite *a* and *b* are given their corresponding Greek letters, with α opposite side *a*, and β opposite side *b*. Label the angles and sides of the triangle shown with these letters.

What is the connection between sides *a*, *b* and *c*?



Complete the following using sides of the triangle:

$\cos\alpha =$ $\qquad\qquad$ $\cos\beta =$

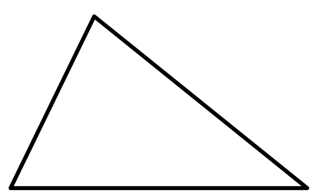
$\sin\alpha =$ $\qquad\qquad$ $\sin\beta =$

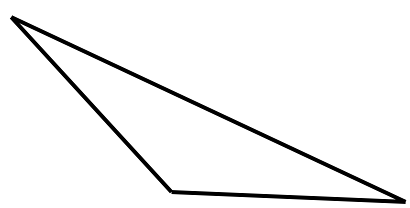
$\tan\alpha =$ $\qquad\qquad$ $\tan\beta =$

Part 2. Oblique Triangles

An **oblique triangle** is a **triangle** with no right angle. An **oblique triangle** has either three acute angles, or one obtuse angle and two acute angles. None of the connections above apply in an oblique triangle. We do have the convention that the angles are labeled with the first three letters of the Greek alphabet and the sides opposite them with their corresponding English alphabet letter.

1. Label the sides and angles of the triangles below.





2. In the triangle above, sketch an altitude through β.

3. Label the altitude *h*.

4. The altitude creates two right triangles inside the original triangle. Notice that α is contained in one of the right triangles, and γ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin α, and one involving sin γ.

$\sin\alpha =$ $\qquad\qquad$ $\sin\gamma =$

5. Notice that these equations both contain *h*. Solve each equation for *h*.

6. Since both equations are equal to h, in #5, they can be set equal to each other. Set the equations equal to each other to form a new equation.

7. Write an equation equivalent to the equation in #5, regrouping a with $\sin \alpha$ and c with $\sin \gamma$.

8. Now go back to the triangle and sketch an altitude that goes through α.

9. Label the altitude j.

10. The altitude again creates two right triangles inside the original triangle. Notice that β is contained in one of the right triangles, and γ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin \beta$, and one involving $\sin \gamma$.

$\sin \beta =$ $\sin \gamma =$

11. Notice that these equations both contain j. Solve each equation for j.

12. Since both equations are equal to j, in #11, they can be set equal to each other. Set the equations equal to each other to form a new equation.

13. Write an equation equivalent to the equation in #12, regrouping b with $\sin \beta$ and c with $\sin \gamma$.

The Law of Sines

Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following ratios hold

________ = ________ = ________

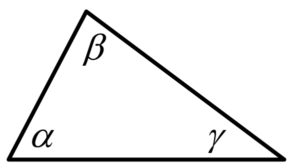
Or equivalently,

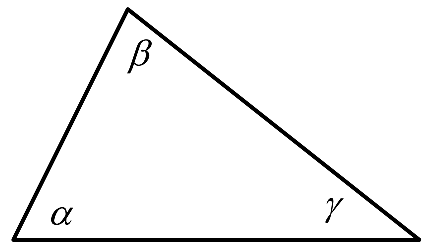
________ = ________ = ________

Part 3. Various Triangles With Three Known Pieces

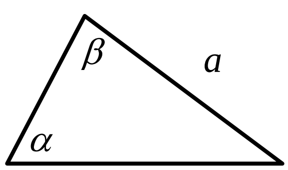
When we are given a triangle and three pieces of information, one of 6 cases may occur.

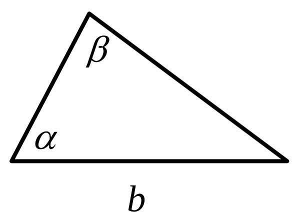
Case 1. AAA You are given 3 angles. What is $\alpha+\beta+\gamma$?
So really, having only 2 angles, you could get the third. This is NOT enough information to determine the sides of the triangle. Recall that similar triangles have congruent angles and their sides are proportional.

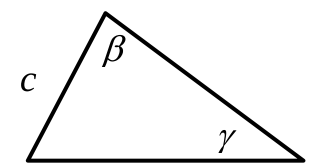




Case 2. AAS (also SAA) You are given two angles and a side that is not between the two angles (called non-included side). Can you find the third angle?
In this case we have an angle and a side opposite that angle. In the first triangle drawn that would be α and *a*. These are called an angle-side opposite pair and can be written as a pair like this (α, a) . We can use the Law of Sines to solve these triangles. Circle each angle-side opposite pair





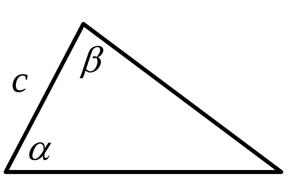


Solve each triangle above given the information below. Triangles are NOT drawn to scale. Round to 3 decimal places.
In the first triangle: $\alpha=13°$, $\beta=17°$, $a=5$

In the second triangle: $\alpha=73.2°$, $\beta=54.1°$, $b=99$

In the third triangle: $\beta=56.3°$, $\gamma=6.7°$, $c=5.9$

Case 3. ASA You are given two angles and a side that is between the two angles (called the included side). To solve this triangle using the Law of Sines, we'd need an angle-side opposite pair and we don't have one. Can you see how to get one though?



Solve the triangle if $\alpha=45°$, $\beta=15°$, $c=7$. See if you can find side *a* WITHOUT using your calculator getting an answer in exact form. Then use your calculator to approximate side *b*.

Case 4. ASS (also SSA) You are given two sides and an angle NOT included between the two sides. You do have angle-side opposite pair BUT this is called the Ambiguous Case and we will see why.

1. Use a protractor to make a 30° angle using the first line given as the initial side. That's the angle. Now along the side just drawn, mark off 2 inches and put a dot for a vertex of the triangle. There is the first side which measures 2. Now from that vertex, make a line 1 inch long that joins the line you were given. So the second side is 1. Solve this triangle you drew without using a calculator.

2. Now use a protractor and make a 30° angle using the second line given as the initial side. There's the angle. Now along the side just drawn, mark off 3 inches and put a dot for the vertex of the triangle. There is the first side which measures 3. Now from that vertex, make a line 1 inch long that joins the line you were given. So the second side is 1. What do you find?

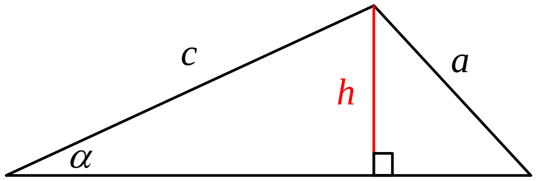
3. Now use a protractor and make a 30° angle using the third line given as the initial side. There's the angle. Now along the side just drawn, mark off 2 inches and put a dot for the vertex of the triangle. There is the first side which measures 2. Now from that vertex, make a line 1.5 inches long that joins the line you were given. So the second side is 1.5. What do you find?

Does your neighbor's triangle look like yours?

Does your teacher's triangle look like yours?

Solve the two triangles where $\alpha = 30°,\ a = 1.5,\ c = 2$

In an ASS triangle, given a, α, and c as shown, find the altitude h in terms of a trig function and the given information.



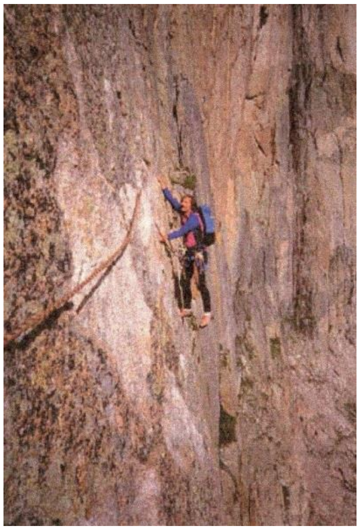
Summary of the Ambiguous Case When Given ASS

If	No triangle exists
If	Then $\gamma = 90°$ and can be solved using right triangle
If	Then two distinct triangles exist (one with γ acute and one with γ obtuse)
If	Then γ is acute and exactly one triangle exists

We will visit the other two cases, namely SAS and SSS later when we do Law of Cosines.

Part 4. Applying the Law of Sines

A rock climber is part way up a climb when he can see both the peak and the base of a mountain across from him. The angle of elevation to the peak is 42° and the angle of depression to the base of the mountain is 36°. If the climber knows the mountain is 2000 feet high, how high is the climber from ground level (where the base of the mountain is)?

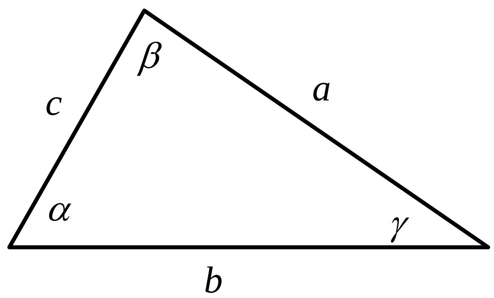


Part 5. Area of a Triangle

We now want to determine the area of a triangle. If we are lucky enough to have a right angle, we know the area.

Area of a right triangle =

But what if it is NOT a right triangle. We consider a triangle below.

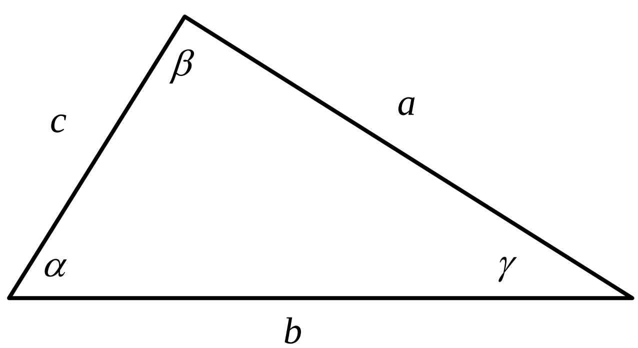


Draw an altitude from the upper vertex down to side *b* and label its length *h*.

What trig function would connect *h* with α and *c*? Write this connection and solve for *h*.

Now express the area of this triangle with the familiar formula you wrote above, with base *b* and replace the *h* with the expression you found above.

We could have drawn the altitude from any of the three vertices. Use the triangle below to draw an altitude from the vertex near α to side *a*. Write two more expressions for h, one using sin β and one using sin γ.



Write three formulas for the area of the triangle above, each using the sine of a different angle.