Name______________________________________

Activity – Verifying Identities

As you complete each proof, sometimes called "establishing" or "verifying" identities, be sure to write down each step as well as explain each step with a rationale or justification of what you did in that step.

Hints:

- Work with the hardest side of the problem and simplify to make it look like the other side. Sometimes it helps to work on both sides, simplifying them down until they match.
- Consider your goal as you simplify.
- It often helps to write all functions in terms of sine and cosine.
- Consider multiplying binomials when they occur.
- Watch for factors, both greatest common factors and factoring a trinomial into two binomials. Don't forget difference of squares and other special factoring patterns.
- Consider multiplying by a conjugate.
- Break fractions into two or more smaller fractions (separate terms in numerator, keep same denominator for each one)
- Find common denominators and combine fractions together.

Example:

Prove that

$$\frac{\sin^3(x) + \sin(x)\cos^2(x)}{\cos(x)} = \tan(x)$$

Let's simplify the left side, because it is more complicated.

Equivalent Statement	**Rationale**
$\frac{\sin^3(x) + \sin(x)\cos^2(x)}{\cos(x)}$	LHS (Left Hand Side)
$= \frac{\sin(x)\,[\sin^2(x) + \cos^2(x)]}{\cos(x)}$	Factor sin(x) out of the numerator.
$= \frac{\sin(x)}{\cos(x)}$	By the fundamental Pythagorean identity, $\sin^2(x) + \cos^2(x) = 1$ (substitute 1 in for $\sin^2(x) + \cos^2(x)$)
$= \tan(x)$	Basic trig identity for tangent. RHS (Right hand side) ✓

1) Prove that

$$\cot(x) + \sin(x) = \frac{1 + \cos(x) - \cos^2(x)}{\sin(x)}$$

Choose the left hand side to “simplify” with algebraic steps and basic trig identities. The final simplified form should match the right hand side. (In this problem, the steps have been done for you. You just have to explain.)

Equivalent Statement	Rationale for each step
LHS: $\cot(x) + \sin(x)$	
$= \dfrac{\cos(x)}{\sin(x)} + \sin(x)$	
$= \dfrac{\cos(x)}{\sin(x)} + \sin(x)\dfrac{\sin(x)}{\sin(x)}$	
$= \dfrac{\cos(x) + \sin^2(x)}{\sin(x)}$	
$= \dfrac{\cos(x) + (1 - \cos^2(x))}{\sin(x)}$	
$= \dfrac{1 + \cos(x) - \cos^2(x)}{\sin(x)}$ **RHS ✓**	

2) Prove that

$$\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

Now it is your turn to show the entire process. First, choose a side to simplify and write it down. Then explain each step as you simplify it to look like the other side.

Equivalent Statement	Rationale

3) Prove that

$$(1+\cot\alpha)^2 - 2\cot\alpha = \frac{1}{(1-\cos\alpha)(1+\cos\alpha)}$$

It is possible to prove an identity by working separately with the two sides until they can be shown to be equivalent to the same expression. Please fill in the missing steps in the following proof.

Equivalent Statement	Rationale	Equivalent Statement	Rationale
$(1+\cot\alpha)^2 - 2\cot\alpha$	**LHS**	$\frac{1}{(1-\cos\alpha)(1+\cos\alpha)}$	**RHS**
$= 1 + 2\cot\alpha + \cot^2\alpha - 2\cot\alpha$		$= \frac{1}{1-\cos^2\alpha}$	
$= \cot^2\alpha + 1$		$= \frac{1}{\sin^2\alpha}$	
$= \csc^2\alpha$		$= \csc^2\alpha$	

By the transitive property, since

$$(1+\cot\alpha)^2 - 2\cot\alpha = \csc^2\alpha$$

and

$$\frac{1}{(1-\cos\alpha)(1+\cos\alpha)} = \csc^2\alpha$$

Then

$$(1+\cot\alpha)^2 - 2\cot\alpha = \frac{1}{(1-\cos\alpha)(1+\cos\alpha)}$$

✓