Activity – Curve Sketching

In this activity we will combine our knowledge of algebra, trigonometry, and calculus to analyze and sketch the graph of a function.

Guidelines for sketching a curve $y = f(x)$ by hand:

    a.   Determine the domain of f.
    b.   Find any intercepts of f, if possible.
    c.   Test for symmetry or periodicity.
    d.   Find any asymptotes of f.
    e.   Using the first derivative find critical values and make a sign chart.
    f.   Determine the intervals of increase and decrease.
    g.   Locate extreme points.
    h.   Using the second derivative, find all critical numbers of f' and make a sign chart.
    i.   Determine intervals of concavity.
    j.   Locate inflection points.
    k.   Sketch the curve.

Problem: Given the function $f(x) = x^3 - 12x$

    a)   Find the domain of f.

    b)   List all intercepts of f as (x, y) ordered pairs.

    c)   Test for symmetry by determining if f is an even or odd function. Would this function be periodic?

    d)   Find the equations of any asymptotes of f. This includes vertical, horizontal, and oblique asymptotes. If the end behavior is not asymptotic, what type of end behavior will you see?

    e)   Find the first derivative, $f'(x)$. Determine any critical values of $f'(x)$. Determine where this derivative is positive and negative. It is helpful to factor the function. In this case, the graph of $y = f'(x)$ is familiar and will quickly tell you the sign values of the function.

    f)   On what interval(s) is $f(x)$ increasing? Decreasing?

g) List any extreme points of f as (x, y) ordered pairs.

h) Find the second derivative, $f''(x)$. Determine any critical values of $f''(x)$. Determine where this derivative is positive and negative. It is helpful to factor the function. Again, the graph of $y = f''(x)$ is familiar and will quickly tell you the sign values of the function.

i) On what interval(s) is $f(x)$ concave up? Concave down?

j) List any inflection points of f as (x, y) ordered pairs.

k) Sketch the graph of the function $f(x)$, labeling any points of interest that were found in the preceding steps.
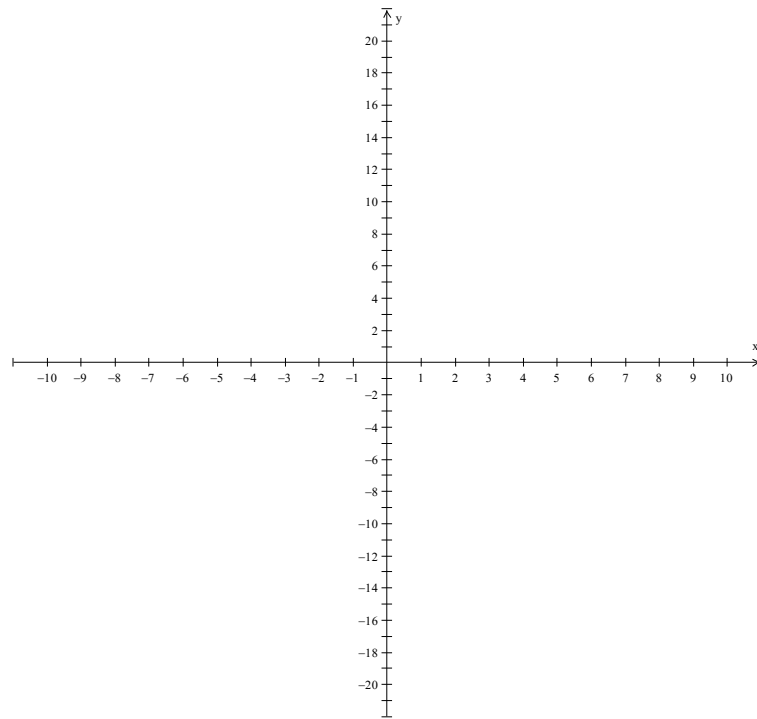
On a separate paper, for each of the following functions, use the steps above to analyze and sketch the curve. You must show work for each of the steps a-k. Note that it will help to simplify your derivatives as much as possible and factor them if possible before attempting to determine their sign values.

1. $f(x) = \dfrac{x^2}{x^2 - 9}$

2. $g(\theta) = \theta + 2\cos\theta$ on $[-2\pi, 2\pi]$

3. $h(x) = x - 3x^{2/3}$

4. $F(x) = x\sqrt{4 - x^2}$ where $F'(x) = \dfrac{4 - 2x^2}{\sqrt{4 - x^2}}$ and $F''(x) = \dfrac{2x^3 - 12x}{(4 - x^2)^{3/2}}$