Name____

Activity - Differential Equations

Part 1. The position of a moving object is given by the function $s(t)$, where s is measured in feet and t in seconds. We determine that the velocity is $v(t) = 4t + 1$ feet per second.

(a) How much does the position change over the time interval $[0,4]$?

(b) Does this give you enough information to determine $s(4)$, the position at time $t = 4$? If so, what is $s(4)$? If not, what additional information would you need to know to determine $s(4)$?

(c) Suppose you are told that the object's initial position $s(0) = 7$. Determine $s(2)$, the object's position 2 seconds later.

(d) If you are told instead that the object's initial position is $s(0) = 3$, what is $s(2)$?

(e) If we only know the velocity $v(t) = 4t + 1$, is it possible that the object's position at all times is $s(t) = 2t^2 + t - 4$? Explain how you know.

(f) Are there other possibilities for $s(t)$? If so, what are they?

(g) If, in addition to knowing the velocity function is $v(t) = 4t + 1$, we know the initial position $s(0)$, how many possibilities are there for $s(t)$?

Part 2.

Express the following statements as differential equations. In each case, you will need to introduce notation to describe the important quantities in the statement so be sure to clearly state what your notation means.

(a) The population of a town grows continuously at an annual rate of 1.25%.

(b) A radioactive sample loses 5.6% of its mass every day.

(c) You have a bank account that continuously earns 4% interest every year. At the same time, you withdraw money continually from the account at the rate of $1000 per year.

(d) A cup of hot chocolate is sitting in a 70° room. The temperature of the hot chocolate cools continuously by 10% of the difference between the hot chocolate's temperature and the room temperature every minute.

(e) A can of cold soda is sitting in a 70° room. The temperature of the soda warms continuously at the rate of 10% of the difference between the soda's temperature and the room's temperature every minute.

Part 3.

Consider the differential equation

$$\frac{dv}{dt} = 1.5 - 0.5v.$$

Which of the following functions are solutions of this differential equation?

(a) $v(t) = 1.5t - 0.25t^2$.

(b) $v(t) = 3 + 2e^{-0.5t}$.

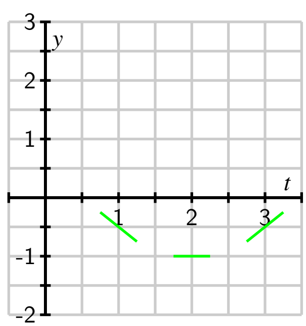
(c) $v(t) = 3$.

(d) $v(t) = 3 + Ce^{-0.5t}$ where C is any constant.

Part 4. Let's consider the initial value problem

$$\frac{dy}{dt} = t - 2, \quad y(0) = 1.$$

(a) Use the differential equation to find the slope of the tangent line to the solution $y(t)$ at $t = 0$. Then use the initial value to find the equation of the tangent line at $t = 0$. Sketch this tangent line over the interval $-0.25 \leq t \leq 0.25$ on the axes provided.



(b) Also shown in the given figure are the tangent lines to the solution $y(t)$ at the points $t = 1, 2,$ and 3 (we will see how to find these later). Use the graph to measure the slope of each tangent line and verify that each agrees with the value specified by the differential equation.

(c) Using these tangent lines as a guide, sketch a graph of the solution $y(t)$ over the interval $0 \leq t \leq 3$ so that the lines are tangent to the graph of $y(t)$.

(d) Use the Fundamental Theorem of Calculus to find $y(t)$, the solution to this initial value problem.

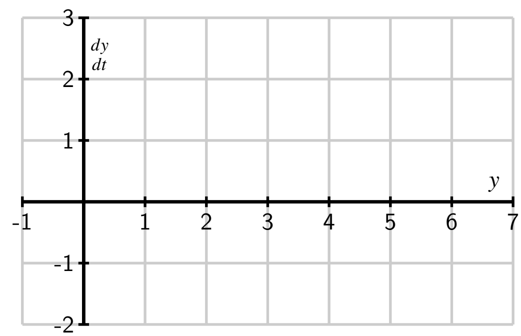
(e) Graph the solution you found in (d) on the axes provided, and compare it to the sketch you made using the tangent lines.

Part 5.

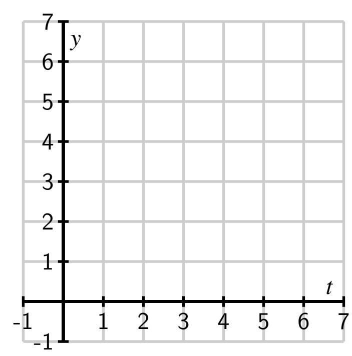
Consider the autonomous differential equation

$$\frac{dy}{dt} = -\frac{1}{2}(y - 4).$$

(a) Make a plot of $\frac{dy}{dt}$ versus y on the axes provided. Looking at the graph, for what values of y does y increase and for what values of y does y decrease?



(b) Next, sketch the slope field for this differential equation on the axes provided.



(c) Use your work in (b) to sketch the solutions that satisfy $y(0) = 0$, $y(0) = 2$, $y(0) = 4$ and $y(0) = 6$.

(d) Verify that $y(t) = 4 + 2e^{-t/2}$ is a solution to the given differential equation with the initial value $y(0) = 6$. Compare its graph to the one you sketched in (c).

(e) What is special about the solution where $y(0) = 4$?