

**Activity - Linearization****Part 1.**

Suppose it is known that for a given differentiable function  $y = g(x)$ , its local linearization at the point where  $a = -1$  is given by  $L(x) = -2 + 3(x + 1)$ .

- (a) Compute the values of  $L(-1)$  and  $L'(-1)$ .
- (b) What must be the values of  $g(-1)$  and  $g'(-1)$ ? Why?
- (c) Do you expect the value of  $g(-1.03)$  to be greater than or less than the value of  $g(-1)$ ? Why?
- (d) Use the local linearization to estimate the value of  $g(-1.03)$ .
- (e) Suppose that you also know that  $g''(-1) = 2$ . What does this tell you about the graph of  $y = g(x)$  at  $a = -1$ ?
- (f) For  $x$  near  $-1$ , sketch the graph of the local linearization  $y = L(x)$  as well as a possible graph of  $y = g(x)$  on the axes provided in Figure.

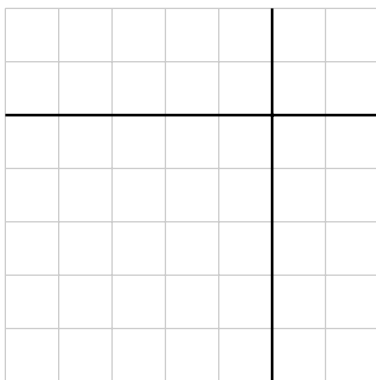


Figure: Axes for plotting  $y = L(x)$  and  $y = g(x)$ .

## Part.2.

This activity concerns a function  $f(x)$  about which the following information is known:

- $f$  is a differentiable function defined at every real number  $x$
- $f(2) = -1$
- $y = f'(x)$  has its graph given in Figure below.

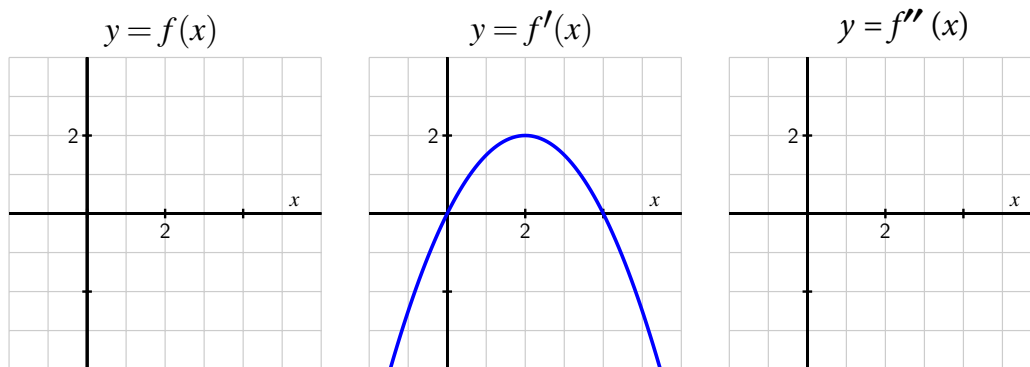


Figure: At center, a graph of  $y = f'(x)$ ; at left, axes for plotting  $y = f(x)$ ; at right, axes for plotting  $y = f''(x)$ .

Your task is to determine as much information as possible about  $f$  (especially near the value  $a = 2$ ) by responding to the questions below.

- Find a formula for the tangent line approximation,  $L(x)$ , to  $f$  at the point  $(2, -1)$ .
- Use the tangent line approximation to estimate the value of  $f(2.07)$ . Show your work carefully and clearly.
- Sketch a graph of  $y = f''(x)$  on the righthand grid in the figure; label it appropriately.
- Is the slope of the tangent line to  $y = f(x)$  increasing, decreasing, or neither when  $x = 2$ ? Explain.
- Sketch a possible graph of  $y = f(x)$  near  $x = 2$  on the lefthand grid in the figure. Include a sketch of  $y = L(x)$  (found in part (a)). Explain how you know the graph of  $y = f(x)$  looks like you have drawn it.
- Does your estimate in (b) over- or under-estimate the true value of  $f(2.07)$ ? Why?