Name_____

Activity - Volumes of Revolution

Part 1. Consider a circular cone of radius 3 and height 5, which we view horizontally as pictured in Figure. Our goal in this activity is to use a definite integral to determine the volume of the cone.

(a) Find a formula for the linear function $y = f(x)$ that is pictured in Figure.

(b) For the representative slice of thickness $\triangle x$ that is located horizontally at a location x (somewhere between $x = 0$ and $x = 5$), what is the radius of the representative slice? Note that the radius depends on the value of x.

(c) What is the volume of the representative slice you found in (b)?

(d) What definite integral will sum the volumes of the thin slices across the full horizontal span of the cone? What is the exact value of this definite integral?

(e) Compare the result of your work in (d) to the volume of the cone that comes from using the formula $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$.

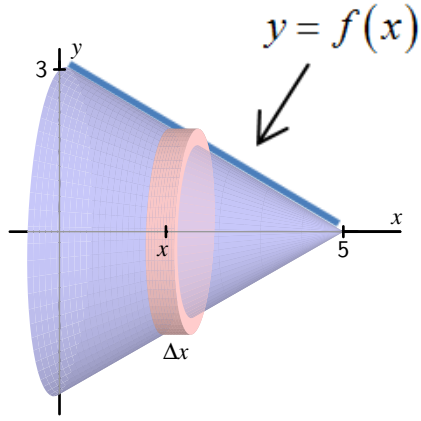

Figure : The circular cone described in Activity

Part 2.

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

(a) The region S bounded by the x-axis, the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the x-axis.

(b) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the x-axis.

(c) The finite region S bounded by the curves $y = \sqrt{x}$ and $y = x^3$; revolve S about the x-axis.

(d) The finite region S bounded by the curves $y = 2x^2 + 1$ and $y = x^2 + 4$; revolve S about the x-axis

(e) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis. How does the problem change considerably when we revolve about the y-axis?

Part 3.

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find.

(a) The region S bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 2$; revolve S about the y-axis.

(b) The region S bounded by the x-axis, the curve $y = \sqrt{x}$, and the line $x = 4$; revolve S about the y-axis.

(c) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the x-axis.

(d) The finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$; revolve S about the y-axis.

(e) The finite region S bounded by the curves $x = (y-1)^2$ and $y = x - 1$; revolve S about the y-axis

Part 4.

In each of the following questions, draw a careful, labeled sketch of the region described, as well as the resulting solid that results from revolving the region about the stated axis. In addition, draw a representative slice and state the volume of that slice, along with a definite integral whose value is the volume of the entire solid. It is not necessary to evaluate the integrals you find. For each prompt, use the finite region S in the first quadrant bounded by the curves $y = 2x$ and $y = x^3$.

(a) Revolve S about the line $y = -2$.

(b) Revolve S about the line $y = 4$.

(c) Revolve S about the line $x = -1$.

(d) Revolve S about the line $x = 5$.