Name_____

Activity - The Notion of a Limit

On a separate sheet of paper, label by part and letter and answer the following questions and then attach to this assignment.

Part 1. Suppose that g is the function given by the graph below. Use the graph to answer each of the following questions.

(a) Determine the values $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, and $g(2)$, if defined. If the function value is not defined, explain what feature of the graph tells you this.

(b) For each of the values $a = -1$, $a = 0$, and $a = 2$, complete the following sentence: "As x gets closer and closer (but not equal) to a, $g(x)$ gets as close as we want to _____."

(c) What happens as x gets closer and closer (but not equal) to $a = 1$? Does the function $g(x)$ get as close as we would like to a single value?

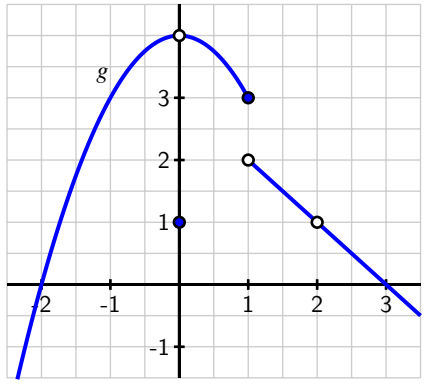


Figure: Graph of $y = g(x)$

Part 2.

Estimate the value of each of the following limits by constructing appropriate tables of values. Then determine the exact value of the limit by using algebra to simplify the function. Finally, plot each function on an appropriate interval to check your result visually.

(a) $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(b) $\lim_{x \to 0} \dfrac{(2 + x)^3 - 8}{x}$

(c) $\lim_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x}$

Part 3. Let $g(x) = -\dfrac{|x+3|}{x+3}$.

(a) What is the domain of g?

(b) Use a sequence of values near $a = -3$ to estimate the value of $\lim_{x\to -3} g(x)$, if you think the limit exists. If you think the limit doesn't exist, explain why.

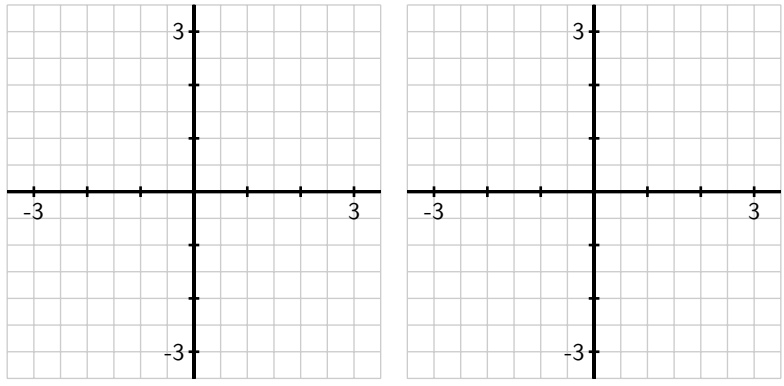
(c) Use algebra to simplify the expression $\frac{|x+3|}{x+3}$ and hence work to evaluate $\lim_{x\to -3} g(x)$ exactly, if it exists, or to explain how your work shows the limit fails to exist. Discuss how your findings compare to your results in (b). (**Hint**: $|a| = a$ whenever $a \geq 0$, but $|a| = -a$ whenever $a < 0$.)

(d) True or false: $g(-3) = -1$. Why?

(e) True or false: $-\frac{|x+3|}{x+3} = -1$. Why? How is this equality connected to your work above with the function g?

(f) Based on all of your work above, construct an accurate, labeled graph of $y = g(x)$ on the interval $[-4,-2]$, and write a sentence that explains what you now know about $\lim\limits_{x\to -3} g(x)$.

Part 4. For each of the following prompts, sketch a graph on the provided axes of a function that has the stated properties.

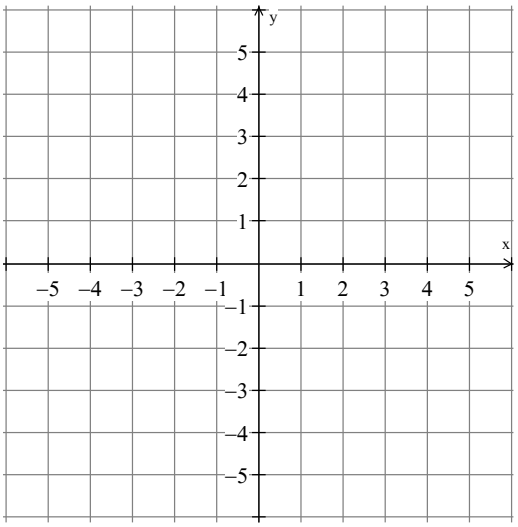


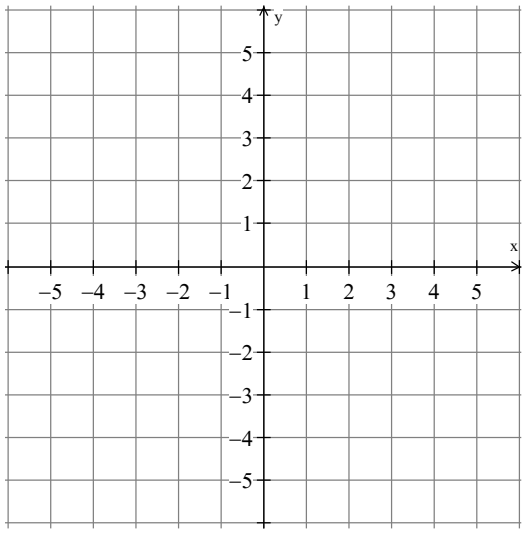
(a) $y = f(x)$ such that

- $f(-2) = 2$ and $\lim\limits_{x\to -2} f(x) = 1$
- $f(-1) = 3$ and $\lim\limits_{x\to -1} f(x) = 3$
- $f(1)$ is not defined and $\lim\limits_{x\to 1} f(x) = 0$
- $f(2) = 1$ and $\lim\limits_{x\to 2} f(x)$ does not exist.

(b) $y = g(x)$ such that

- $g(-2) = 3$, $g(-1) = -1$, $g(1) = -2$, and $g(2) = 3$
- At $x = -2, -1, 1$ and 2, g has a limit, and its limit equals the value of the function at that point.
- $g(0)$ is not defined and $\lim\limits_{x\to 0} g(x)$ does not exist.





(c) $y = h(x)$ such that

- $\lim_{x\to 2} h(x) = \infty$
- $\lim_{x\to -2^+} h(x) = \infty$
- $\lim_{x\to -2^-} h(x) = -\infty$
- $\lim_{x\to -\infty} h(x) = 0$
- $\lim_{x\to \infty} h(x) = 0$
- $f(0) = 0$

(d) $y = k(x)$ such that

- $k(0) = 3$
- $\lim_{x\to 0^-} k(x) = 4$
- $\lim_{x\to 0^+} k(x) = 2$
- $\lim_{x\to -\infty} h(x) = -\infty$
- $\lim_{x\to \infty} h(x) = 3$
- $\lim_{x\to 4^-} h(x) = -\infty$
- $\lim_{x\to 4^+} h(x) = \infty$

Part 5. Using limits and correct limit notation, find all horizontal and vertical asymptotes for the following functions.

(*a*) $f(x) = \dfrac{2x^2 + x - 1}{x^2 + x - 2}$ (*b*) $g(x) = \dfrac{x^3 - x}{x^2 - 6x + 5}$

Part 6.

(a) How many horizontal asymptotes can a function have? List all possibilities.

(b) How many vertical asymptotes can a function have? Why?

(c) Can a function cross a horizontal asymptote? Why?

(d) Can a function cross a vertical asymptote? Why?

(e) Use the Squeeze Theorem to evaluate $\lim_{x\to\infty}\frac{4\sin x}{x}$.

(f) Graph $f(x)=\frac{4\sin x}{x}$. How many times does the graph cross the horizontal asymptote?