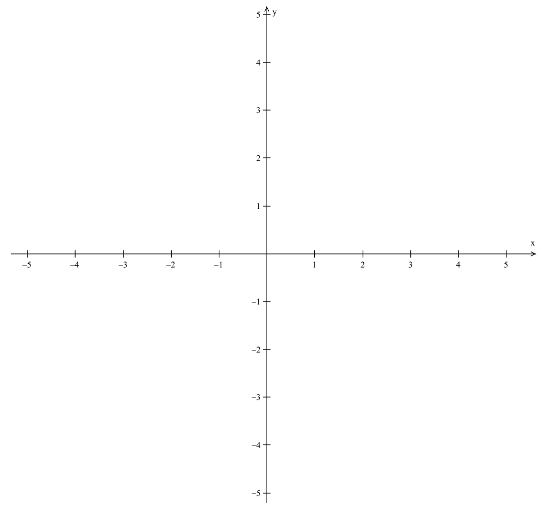
Name_________________________________

Activity – Inverse Trigonometric Functions

Part 1. Reviewing Inverse Functions

a) Recall from college algebra that you learned how to find inverse functions.
Given $f(x) = x^3 + 1$, find $f^{-1}(x)$



b) Graph both $f(x)$ and $f^{-1}(x)$ on the graph given.

c) On the graph, draw a dashed line at $y = x$.
What is the relationship between $f(x)$, $f^{-1}(x)$ and this line?

d) Recall that only one-to-one functions have inverse functions. What does it mean for a function to be one-to-one?

e) Find $\left(f^{-1} \circ f\right)(x)$. Recall that this means $f^{-1}\left(f(x)\right)$.

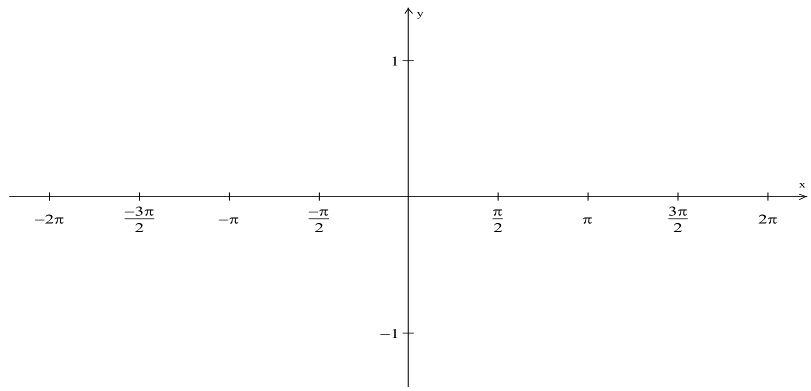
f) Explain what happens when you apply in inverse function to a function.

g) Make a guess as to what the simplification would be if you had $\cos^{-1}\left(\cos\theta\right)$.

Part 2. The Inverse Cosine Function

a) Graph the cosine function.

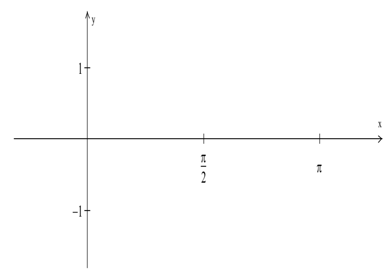
b) Is the function one-to-one? Will it have an inverse function?



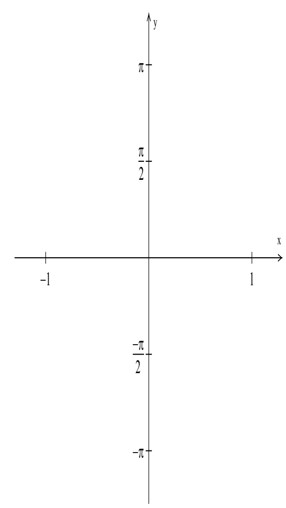
c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f(x) = \cos x$, on $[0, \pi]$.

Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}(x) = \cos^{-1} x$.



f) Find the following:

i) $\cos^{-1} 0$

ii) $\cos^{-1} 1$

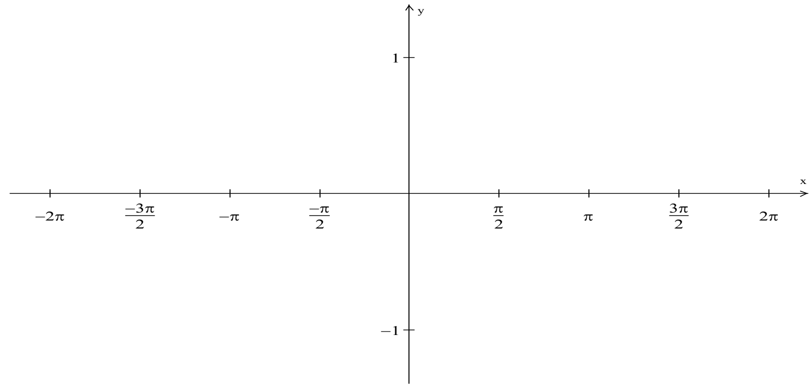
iii) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

iv) $\cos^{-1}\left(-\frac{1}{2}\right)$

v) $\cos^{-1}\left(\frac{1}{2}\right)$

vi) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Part 3. The Inverse Sine Function



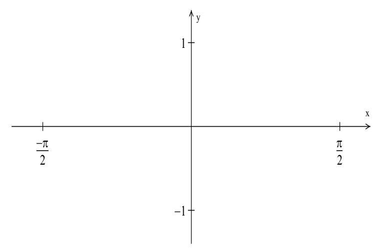
a) Graph the sine function.

b) Is the function one-to-one?
Will it have an inverse function?

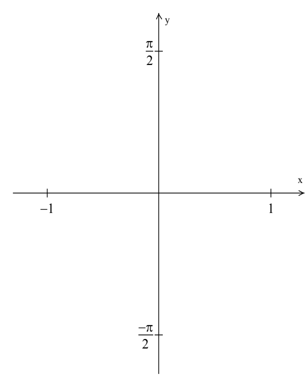
c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f(x) = \sin x,$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}(x) = \sin^{-1} x$.



f) Find the following:

i) $\sin^{-1} 0$

ii) $\sin^{-1} 1$

iii) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

iv) $\sin^{-1}\left(-\frac{1}{2}\right)$

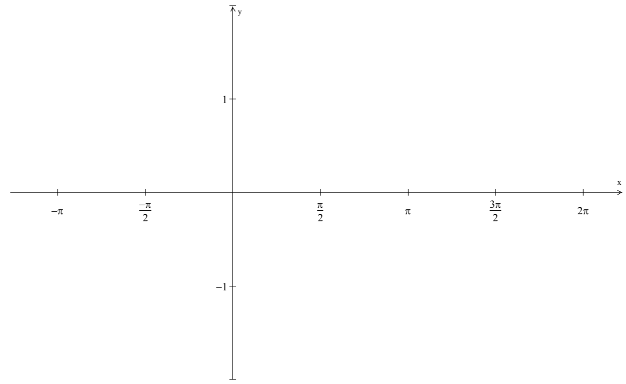
v) $\sin^{-1}\left(\frac{1}{2}\right)$

vi) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Part 4. The Inverse Tangent Function

a) Graph the tangent function.

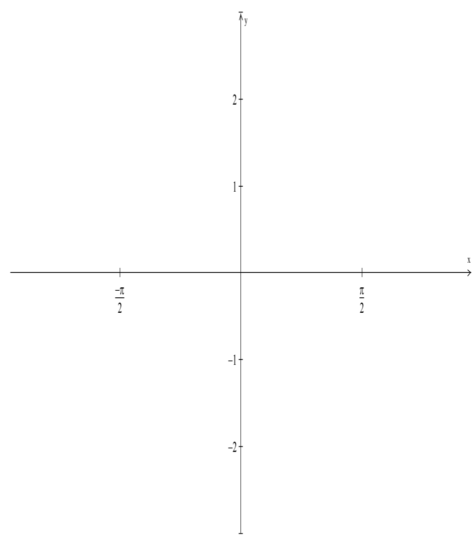
b) Is the function one-to-one?
Will it have an inverse function?



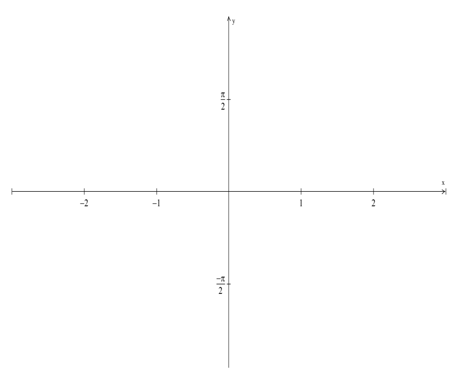
c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f(x) = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Include asymptotes.

Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}(x) = \tan^{-1} x$. Include asymptotes.



f) Find the following:

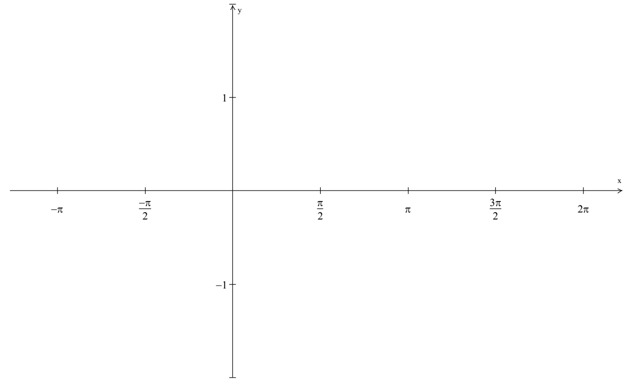
i) $\tan^{-1} 0$

ii) $\tan^{-1} 1$

iii) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

iv) $\tan^{-1}(-1)$

Part 5. The Inverse Cotangent Function

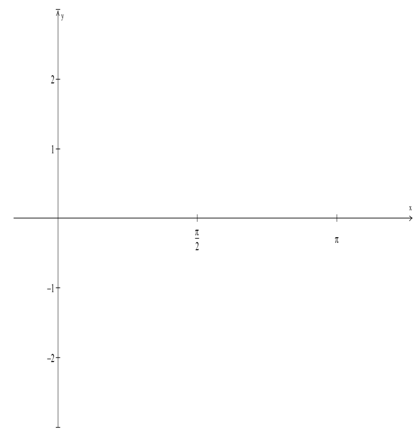


a) Graph the cotangent function.

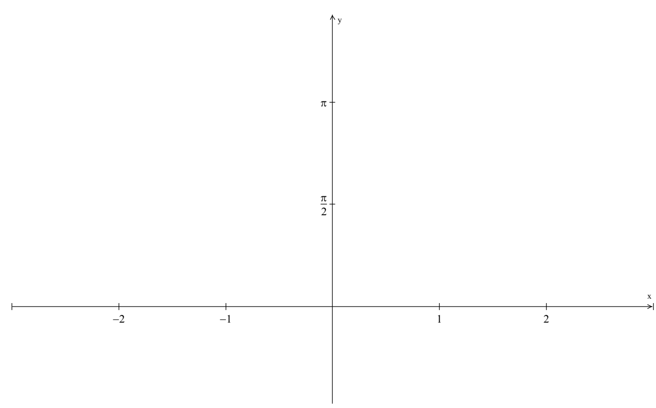
b) Is the function one-to-one?
Will it have an inverse function?

c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f(x) = \cot x$ on $(0, \pi)$. Include asymptotes.
Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}(x) = \cot^{-1} x$. Include asymptotes.

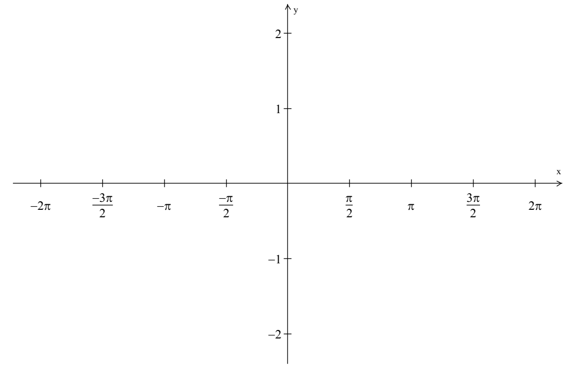


f) Find the following:

i) $\cot^{-1} 0$ ii) $\cot^{-1} 1$

iii) $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ iv) $\cot^{-1}(-1)$

Part 6. The Inverse Secant Function



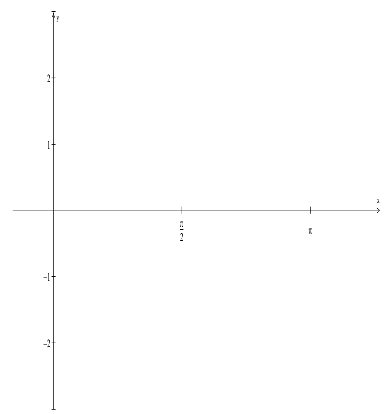
a) Graph the secant function.

b) Is the function one-to-one?
Will it have an inverse function?

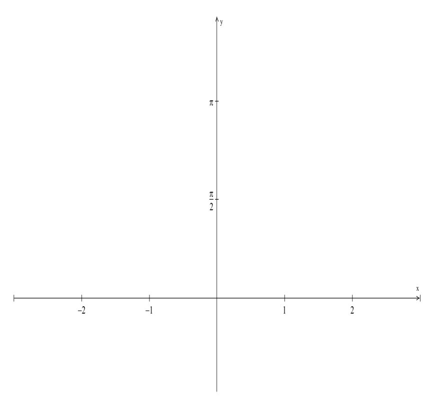
c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f(x) = \sec x$ on $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$. Include asymptote.

Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}(x) = \sec^{-1} x$. Include asymptote.



f) Find the following:

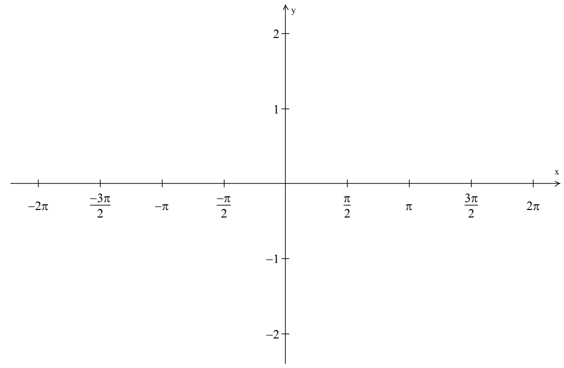
i) $\sec^{-1} 2$

ii) $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

iii) $\sec^{-1}\left(\sqrt{2}\right)$

iv) $\sec^{-1}(-2)$

Part 6. The Inverse Cosecant Function



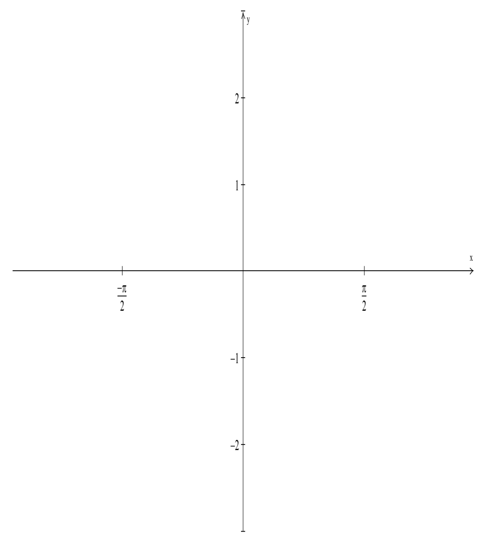
a) Graph the cosecant function.

b) Is the function one-to-one?
Will it have an inverse function?

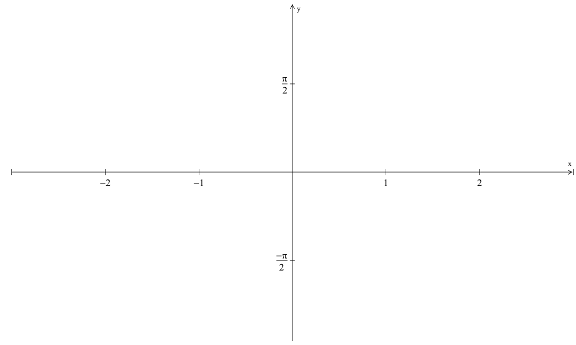
c) Can we restrict the domain (taking only a portion of this function) so that it is one-to-one? If so, write a domain restriction so that we have a one-to-one function.

d) Graph $f\left(x\right)=\csc x$ on $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$. Include asymptote.

Is the function one-to-one?



e) By reflecting the graph above about the line $y = x$ (trading x and y places), graph the inverse function on the given axes. Label this function $f^{-1}\left(x\right)=\sec^{-1}x$. Include asymptote.



f) Find the following:

i) $\csc^{-1}2$

ii) $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

iii) $\csc^{-1}\left(\sqrt{2}\right)$

iv) $\csc^{-1}\left(-2\right)$

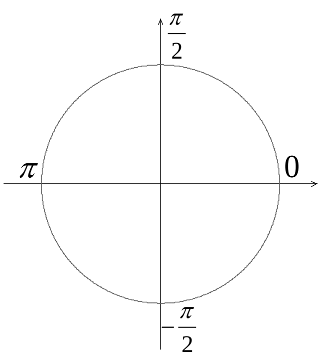
Part 7. Summary and Unit Circle Connection

a) Complete the following tables:

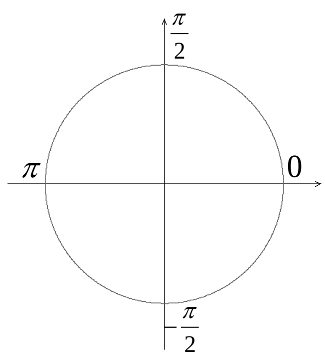
Function	$y = \cos x$	$y = \cos^{-1} x$	$y = \sin x$	$y = \sin^{-1} x$	$y = \tan x$	$y = \tan^{-1} x$
Domain						
Range						

Function	$y = \sec x$	$y = \sec^{-1} x$	$y = \csc x$	$y = \csc^{-1} x$	$y = \cot x$	$y = \cot^{-1} x$
Domain						
Range						

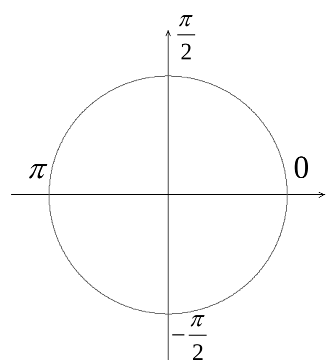
b) On the unit circle, shade the **range** of each inverse trig function.



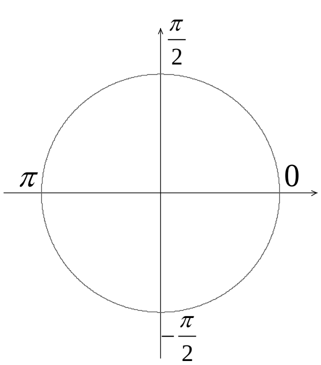
$y = \cos^{-1} x$



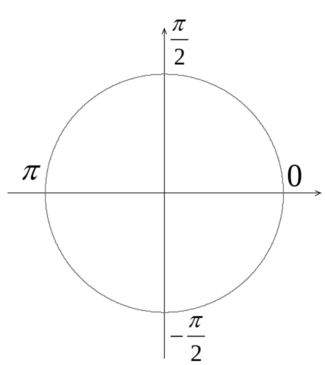
$y = \sin^{-1} x$



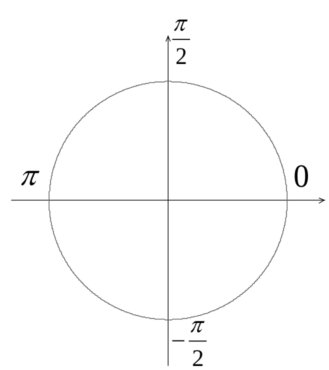
$y = \tan^{-1} x$



$y = \sec^{-1} x$



$y = \csc^{-1} x$



$y = \cot^{-1} x$