

Activity - Derivatives of Inverse Trigonometric Functions

Part 1.

The following prompts in this activity will lead you to develop the derivative of the inverse tangent function.

- (a) Let $y = \arctan(x)$. Use the relationship between the arctangent and tangent functions to rewrite this equation using only the tangent function.
- (b) Differentiate both sides of the equation you found in (a). Solve the resulting equation for y' , writing y' as simply as possible in terms of a trigonometric function evaluated at y .
- (c) Recall that $y = \arctan(x)$. Update your expression for y' so that it only involves trigonometric functions and the independent variable x .
- (d) Introduce a right triangle with angle θ so that $\theta = \arctan(x)$. What are the three sides of the triangle?
- (e) Using the right triangle, what is the value of $\cos(\arctan(x))$?
- (f) Use the results of your work above to find an expression involving only 1 and x for the derivative of $\arctan(x)$.

Part 2.

Following analogous steps from Part 1, find the derivatives of

- (a) $y = \arcsin(x)$
- (b) $y = \arccos(x)$
- (c) $y = \operatorname{arccot}(x)$
- (d) $y = \operatorname{arccsc}(x)$
- (e) $y = \operatorname{arcsec}(x)$