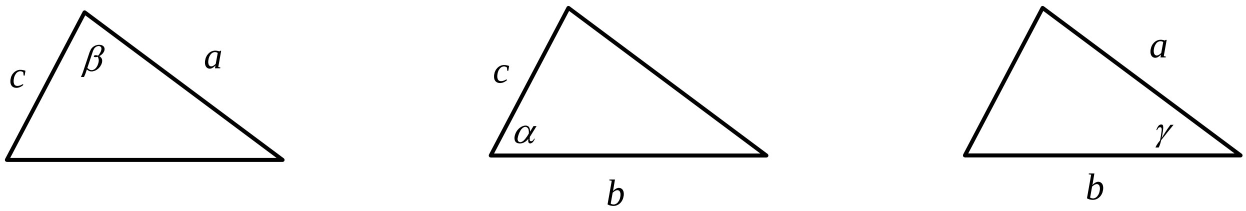
Name______________________________________

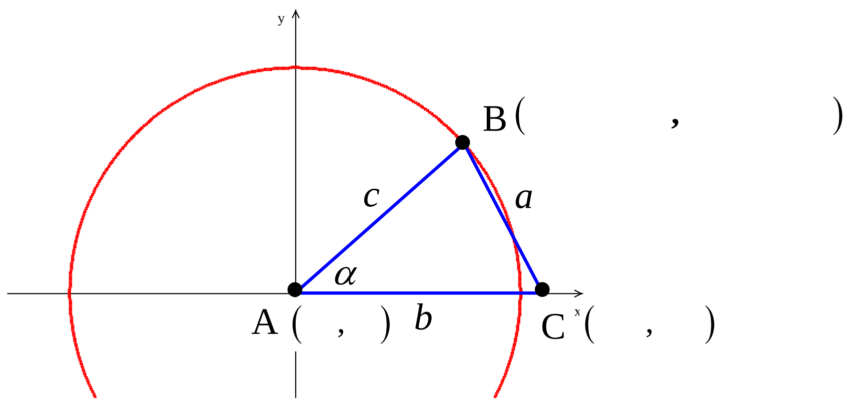
Activity – Law of Cosines

Recall we stated that when we are given a triangle and three pieces of information, one of 6 cases may occur. We will now look at the two remaining cases, namely SAS and SSS.

Case 5. SAS You are given the length of two sides of a triangle and the measure of the angle between those two sides. As you can see, we don't have an angle-side opposite pair so we cannot use the Law of Sines.



Let's look at a circle of radius *c* and a triangle with one side on the *x* axis as shown below.
Fill in the (x, y) coordinates for the vertices of the triangle, Points *A*, *B* and *C*.
Hint: B is in terms of cosine and sine. An altitude of the triangle to the *x* axis may help.



Express the length of side *a* using the distance formula.

Now square both sides and simplify.

With this equation, if we had SAS as in the middle triangle shown above, we could find side *a*.
Solve the triangle with $\alpha = 59.3°$, $b = 12$, $c = 7$. Round to two decimal places.

The triangle and circle picture above can be modified by rotating the triangle so a different angle is at the origin in order to find analogous equations for b^2 and c^2. Complete the following:

The Law of Cosines

Given a triangle with angle-side opposite pairs (α, a), (β, b) and (γ, c), the following equations hold

$a^2 =$ $\qquad\qquad$ $b^2 =$ $\qquad\qquad$ $c^2 =$

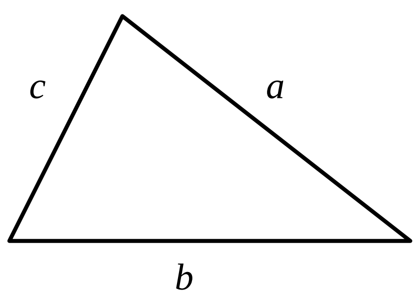
Or solving for cosine in each equation,

$\cos\alpha =$ $\qquad\qquad$ $\cos\beta =$ $\qquad\qquad$ $\cos\gamma =$

NOTE: When using the Law of Cosines, it's best to find the measure of the largest unknown angle first to avoid an ambiguous case.

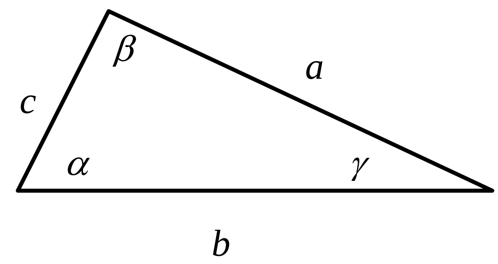
Law of Cosine in Words: The square of one side of a triangle equals the sum of the squares of the squares of the other two sides minus two times the product of the length of those two sides times the cosine of the angle between those two sides.

Case 6. SSS You are given the length of all three sides of a triangle but do not know any angles. Again, we don't have an angle-side opposite pair so we cannot use the Law of Sines. We can however find an angle using the second form of the equations above. We will then need to use an inverse cosine to find the angle. What is the range of angles we could get for inverse cosine (in degrees)?



To solve the triangle: $a = 4,\ b = 7,\ c = 5$, which angle should we find first?
Find all 3 angles.

Angles in a Triangle



The triangle shown is drawn to scale.

Which angle is largest? Which side is longest?

Which angle is smallest? Which side is shortest?

Is there a correlation between angle size and length of the side opposite?

Can you have more than one obtuse angle in a triangle?

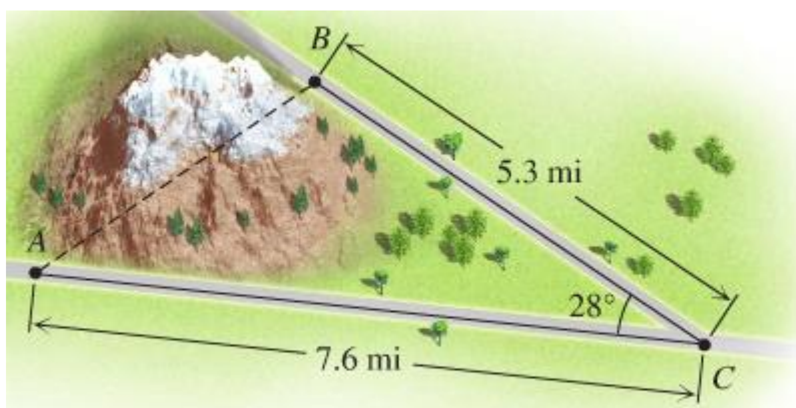
Could you get an obtuse angle for $\sin^{-1}\theta$? How about $\cos^{-1}\theta$?

These are the reasons when using the Law of Cosines with SSS that you find the angle opposite the largest side first. That way if there is an obtuse angle in the triangle, you can find it. Otherwise, when you do Law of Sines you may have an ambiguous case and two possible triangles.

Application Using the Law of Cosines

You want to build a tunnel through the mountain to connect two roads as shown.

Find the distance from Point A to Point B and also the angles to turn from the road to go through the mountain at these points.



Area of a Triangle With SSS

We now can find the area of a right triangle or if we have at least one angle in an oblique triangle, but what if we only have the three sides of the triangle?

Using the Law of Cosines we can derive a formula for finding that area. The derivation is quite involved and can be found in your book at the end of the Law of Cosines section. It is called Heron's Formula, named after Heron of Alexandria who lived from 10-70 AD.

Heron's Formula

Given a triangle with sides *a*, *b* and *c* with $s = \frac{1}{2}(a+b+c)$, the area *A* enclosed by the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

In using Heron's formula we multiply four lengths together. What are the units where you use Heron's Formula? Are these consistent with area?

You read on the Internet that the "Bermuda Triangle" covers an area of more than half a million square miles. The triangle runs from Hamilton, Bermuda to Miami, Florida to San Juan, Puerto Rico so you pull up Google Maps and find the distances between these cities shown below. It the claim accurate?