Name: _____

Activity – Logarithmic Differentiation

Part 1: Properties of Logarithms

Complete the following statements regarding properties of logs:

1. $log_a(MN) =$ _____

   Translation: The log of a product equals the _____ of the logs.

2. $log_a(M/N)$ = _____

   Translation: The log of a _____ equals the _____ of the logs.

3. $log_a M^r =$ _____

   Translation: The log of ___ raised to the ___ power equals ___ times the _____.

Part 2: Logarithm Practice

1. Rewrite each expression as the sum and/or difference of logarithms expressing powers as factors.

   a) $log_2\left(\frac{xy^5}{z}\right)$

   b) $ln\left(\frac{\sqrt{w-8}}{3x}\right)$

2. Rewrite each expression as a single logarithm.

   a) $4\log x + \frac{1}{3}\log y - 10\log z$

   b) $\frac{3}{8}\ln p - 7\ln r - 2\ln e$

Part 3: Differentiation

1. Consider the function $f(x) = x^4 \cdot e^x \cdot sinx$

   Find the derivative of *f* using the product rule (without logs). Recall that multiplication is associative: *abc* is the same as (*ab*)*c* or *a*(*bc*).

   Now find the derivative of *f* using logarithmic differentiation by doing the following: Take the natural log of both sides of the equation.

   Find the derivative with respect to *x* of both sides (implicit differentiation is needed).

   Isolate the derivative of *f* with respect to *x* on one side of the equation by multiplying both sides of the equation by $f(x)$.

   Substitute $x^4 \cdot e^x \cdot sinx$ for $f(x)$.

   Are the results of the two methods for finding the derivative the same?

   Which method is easier?

2. Consider the function $y = (\tan x)^x$

   Is it possible to find the derivative $\frac{dy}{dx}$ by using the power rule? Why or why not?

   Find $\frac{dy}{dx}$ using logarithmic differentiation.

Part 4: Practice

Use logarithmic differentiation to find the derivative of each of the following functions.

a) $g(x) = \sqrt{\frac{x-1}{x^5}}$

b) $f(x) = \frac{2x^3 cos^2 x}{\ln x}$

c) $h(x) = x^{\sin x}$