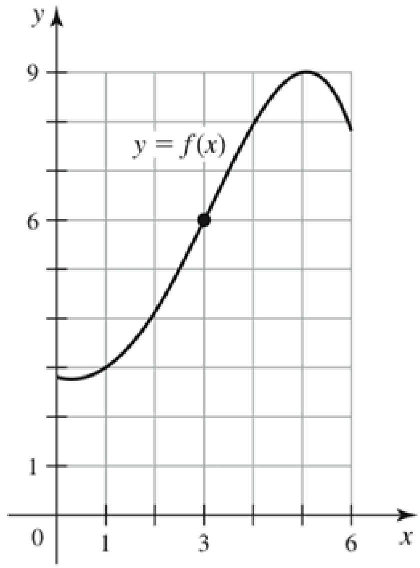
Precise Definition of Limit Graphically

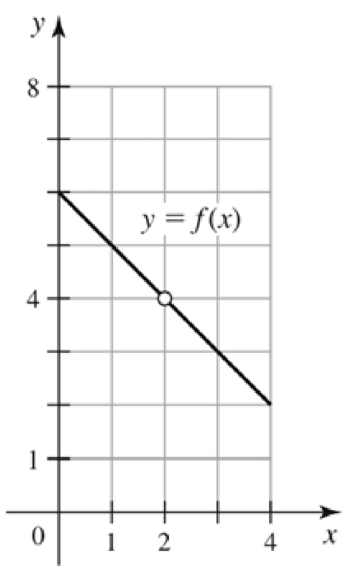
1. **Determining values of δ from a graph** The function f in the figure satisfies $\lim_{x \to 3} f(x) = 6$.

a. If $0 < |x - 3| < \delta$, then $|f(x) - 6| < 3$.
b. If $0 < |x - 3| < \delta$, then $|f(x) - 6| < 1$.



2. **Determining values of δ from a graph** The function f in the figure satisfies $\lim_{x \to 2} f(x) = 4$.

a. If $0 < |x - 2| < \delta$, then $|f(x) - 4| < 1$.
b. If $0 < |x - 2| < \delta$, then $|f(x) - 4| < 1/2$.



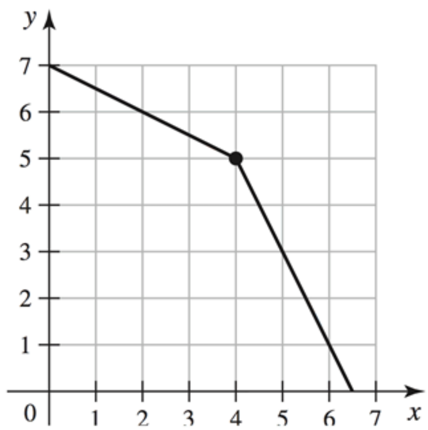
3. **Finding a symmetric interval** The function f in the figure satisfies $\lim_{x\to 4} f(x) = 5$. For each value of ϵ, find a value of $\delta > 0$ such that

$$|f(x) - 5| < \epsilon \quad \text{whenever} \quad 0 < |x - 4| < \delta.$$

a. $\epsilon = 2$

b. $\epsilon = 1$

c. For any $\epsilon > 0$, make a conjecture about the corresponding values of δ



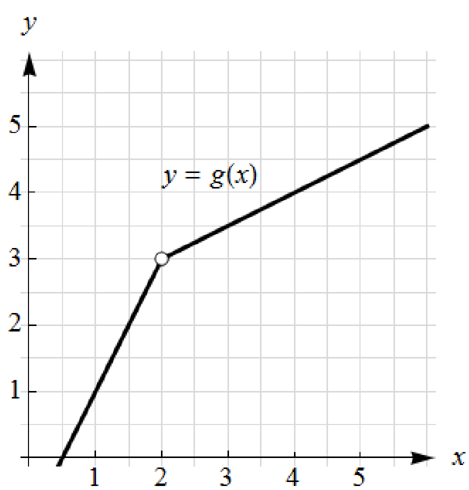
4. **Finding a symmetric interval** The function f in the figure satisfies $\lim_{x\to 2} f(x) = 3$. For each value of ϵ, find a value of $\delta > 0$ such that

$$|f(x) - 3| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta.$$

a. $\epsilon = 2$

b. $\epsilon = 1$

c. For any given value of ϵ, make a conjecture for the corresponding values of δ that satisfy the limit condition.



5. Given $\varepsilon > 0$, find a $\delta > 0$ that could be used with the precise definition of limit to prove that $\lim_{x\to 4}(4x-15)=1$ and then write up a formal proof using the precise definition of limit.