

Activity – Graphs of Cosine and Sine Functions

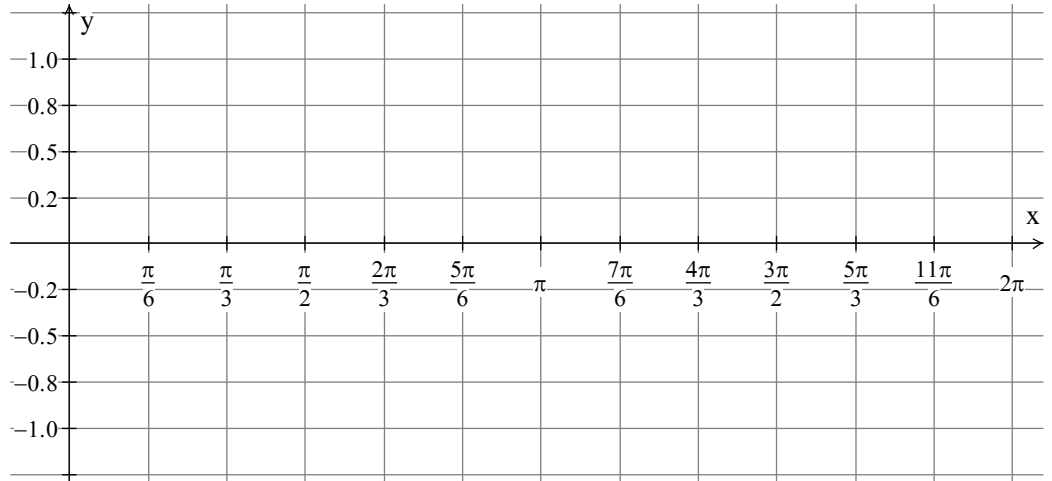
We are interested in the graphs of cosine and sine functions. After finding the basic graphs, we will look at various transformations looking at  $C(x) = A \cos(\omega x + \phi) + B$  and  $S(x) = A \sin(\omega x + \phi) + B$

**Part 1. Basic cosine and sine graphs.**

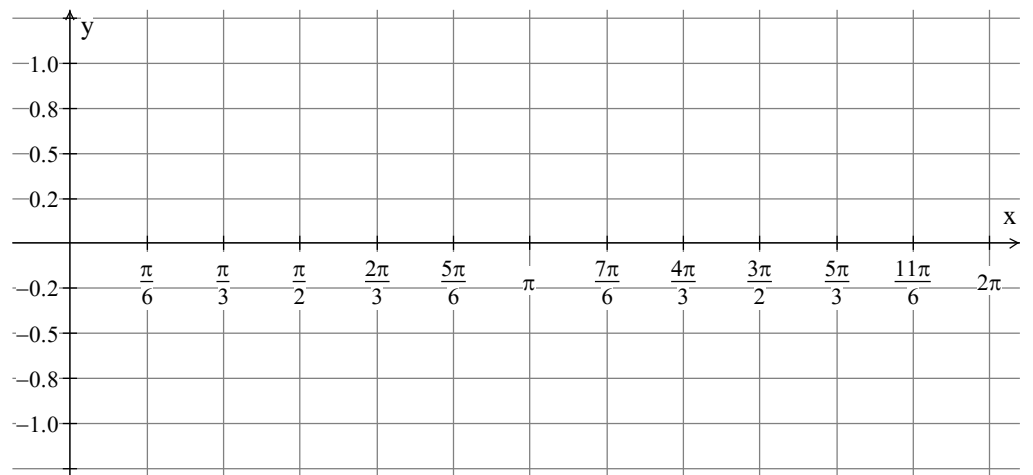
From the unit circle, we know various values of  $\cos\theta$  and  $\sin\theta$ . Complete the table on the left below with these values. For irrational values, also give a one decimal place approximate.

Then plot those values on the Cartesian Plane given to the right.

$x$	$\cos(x)$	$\sin(x)$
0		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$		
$\frac{3\pi}{4}$		
$\frac{5\pi}{6}$		
$\pi$		
$\frac{7\pi}{6}$		
$\frac{5\pi}{4}$		
$\frac{4\pi}{3}$		
$\frac{3\pi}{2}$		
$\frac{5\pi}{3}$		
$\frac{7\pi}{4}$		
$\frac{11\pi}{6}$		
$2\pi$		



$$C(x) = \cos(x)$$



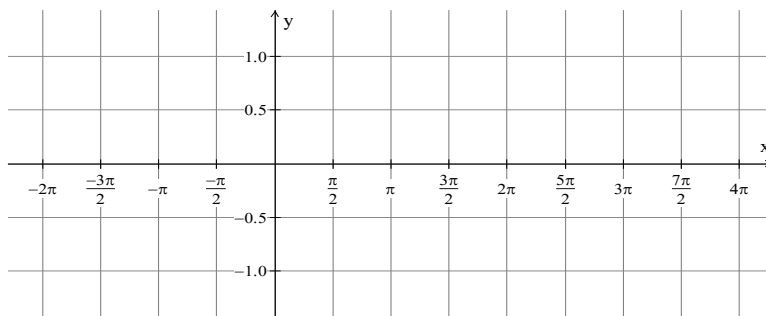
$$S(x) = \sin(x)$$

## Part 2. Extending The Graphs

a) Thinking of coterminal angles and their values, extend the graphs of both cosine and sine on the graphs below.

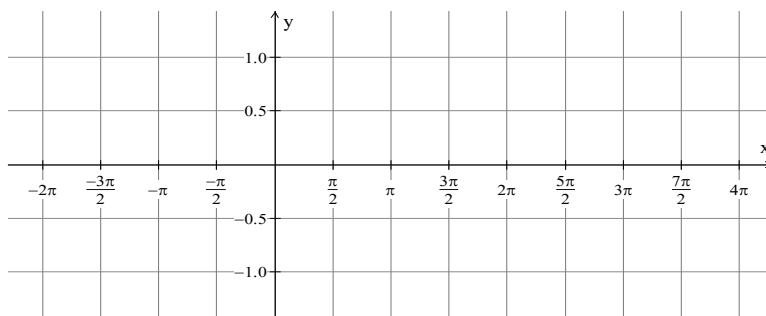
$$C(x) = \cos(x)$$

Is the cosine function even or odd?



$$S(x) = \sin(x)$$

Is the sine function even or odd?

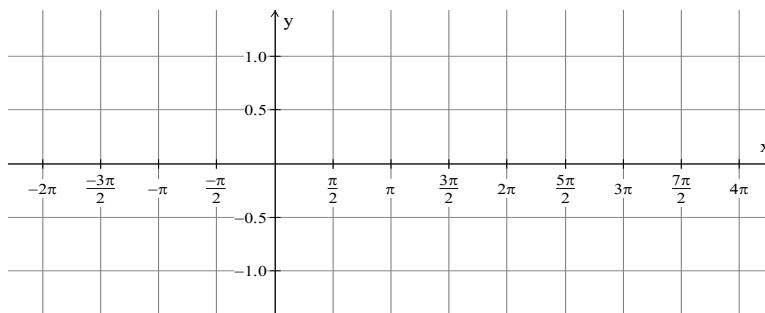


b) Sines and cosines are called **periodic functions** because there is a real number  $c$ , such that  $f(t+c) = f(c)$  for all real numbers  $t$  in the domain. What this means is the function repeats itself. The **period** is the smallest positive number  $p$  for which  $f(t + p) = f(t)$ .

	Period	Domain	Range	$x$ values of maximums	$x$ values of minimums	$x$ - intercepts	$y$ -intercept
$y = \cos(x)$							
$y = \sin(x)$							

f) Graph both  $C(x) = \cos(x)$  and  $S(x) = \sin(x)$  on the same graph.

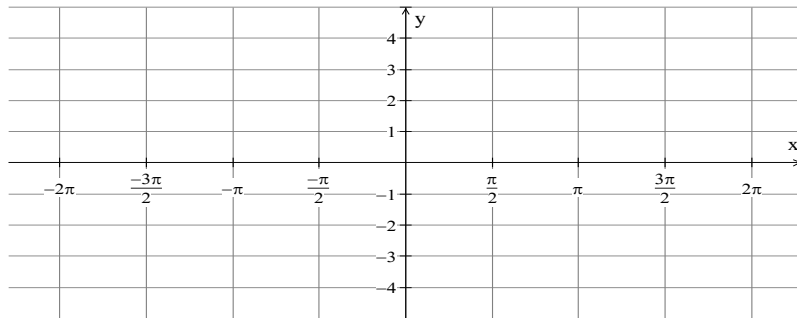
What if you applied a horizontal shift of  $\frac{\pi}{2}$  to the right to your cosine graph?



### Part 3. Transformations of the Sine and Cosine Graphs

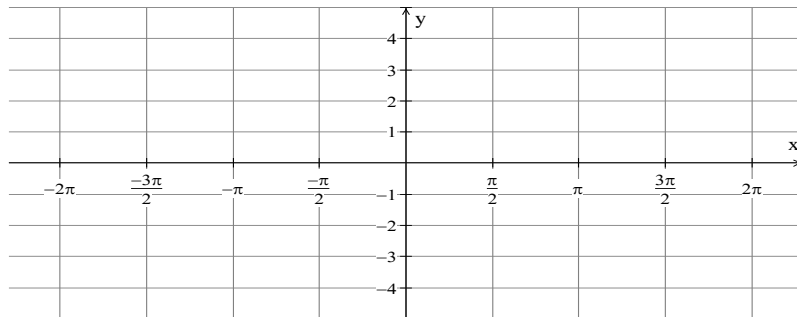
- a) First we will look at  $y = A \sin x$  to determine how changing the value of  $A$  affects the graph.  
Explain how various values of  $A$  affect the graph.

- b) Graph  $y = -3 \sin x$



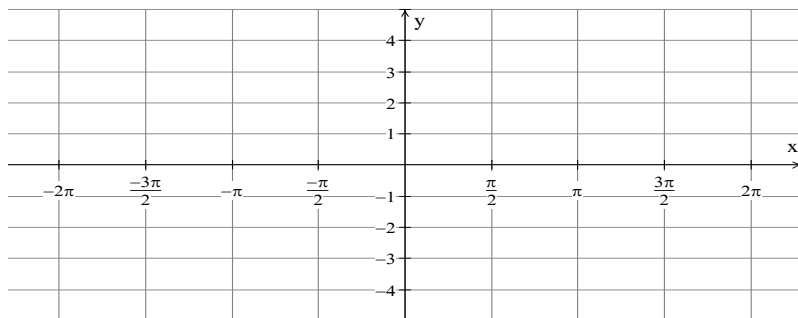
- c) Now we'll explore  $y = \sin \omega x$ . Explain how changing the coefficient on  $x$  changes the graph.

- d) Graph  $y = \sin \omega x$



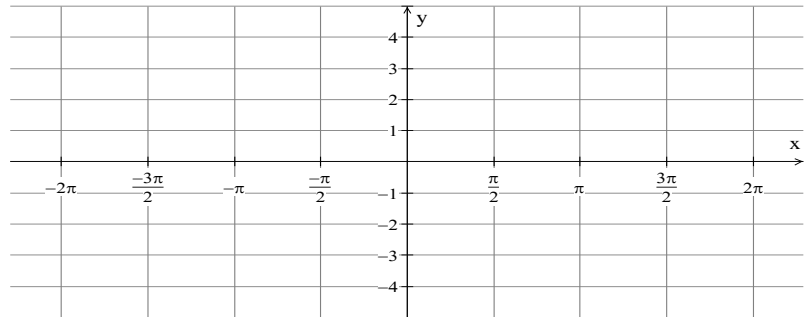
- e) Next let's look at  $y = \sin x + B$ . Explain how adding a constant after the  $\sin x$  affects the graph.

- f) Graph  $y = \sin x - 2$



g) Last let's see  $y = \sin(x + \phi)$ . Explain how adding a constant to the  $x$  affects the graph.

h) Graph  $y = \sin\left(x - \frac{\pi}{4}\right)$



i) Finally, let's put it all together and graph  $y = -2\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$ .

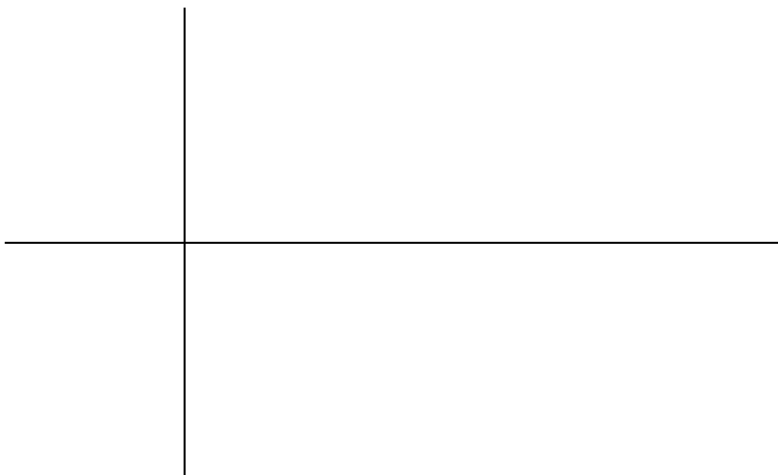
i) What is the period of this trig function?

ii) What is the amplitude of the graph?

iii) What is the phase shift?

iv) What is the vertical shift?

1. Make 4 tick marks on your graph and label the 4<sup>th</sup> one with the period. Then label each of the other tick marks by dividing the period by 4.
2. If there is a vertical shift, draw a dotted baseline that your sine function will oscillate about.
3. Mark your y axis to include  $\pm$  the amplitude from your baseline.
4. Mark the 4 key points for a sine graph with the period, amplitude and vertical shift above. Don't forget to go down instead of up first if there is a negative on the  $A$ .
5. Now apply the phase shift to each point and join them up in a sinusoidal curve.



## Part 4. Transforming Cosine Graphs

The transformations you learned for sine graphs also apply to cosine graphs. The only thing different is that at 0, the cosine starts at its maximum (unless reflected, in which case it will start at its minimum).

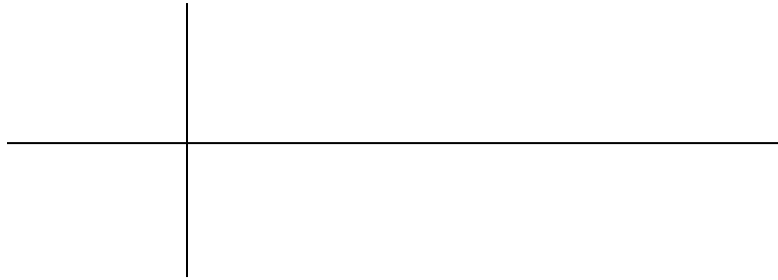
a) Graph  $y = 3\cos(2x)$

i) Period?

ii) Vertical shift?

iii) Amplitude?

iv) Phase shift?



b) Graph  $y = \frac{1}{2}\cos(3x)$

i) Period?

ii) Vertical shift?

iii) Amplitude?

iv) Phase shift?



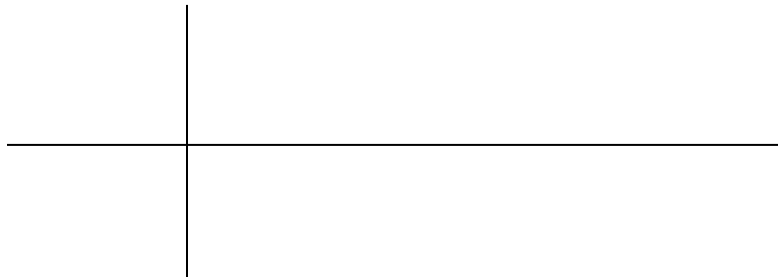
c) Graph  $y = \cos\left(2x + \frac{\pi}{3}\right)$

i) Period?

ii) Vertical shift?

iii) Amplitude?

iv) Phase shift?



d) Graph  $y = -\cos\left(x - \frac{\pi}{2}\right) + 2$

i) Period?

ii) Vertical shift?

iii) Amplitude?

iv) Phase shift?



## Part 5. Finding Equations of Sinusoidal Curves

For each of the graphs below:

- Find a cosine function  $C(x) = A\cos(\omega x + \phi) + B$
- Find a sine function  $S(x) = A\sin(\omega x + \phi) + B$

1.



2.

