Name____________________________________

Activity - Work

Part 1. A bucket is being lifted from the bottom of a 50-foot deep well; its weight (including the water), B, in pounds at a height h feet above the water is given by the function $B(h)$. When the bucket leaves the water, the bucket and water together weigh $B(0) = 20$ pounds, and when the bucket reaches the top of the well, $B(50) = 12$ pounds. Assume that the bucket loses water at a constant rate (as a function of height, h) throughout its journey from the bottom to the top of the well.

(a) Find a formula for $B(h)$.

(b) Compute the value of the product $B(5)\triangle h$, where $\triangle h = 2$ feet. Include units on your answer. Explain why this product represents the approximate work it took to move the bucket of water from $h = 5$ to $h = 7$.

(c) Is the value in (b) an over- or under-estimate of the actual amount of work it took to move the bucket from $h = 5$ to $h = 7$? Why?

(d) Compute the value of the product $B(22)\triangle h$, where $\triangle h = 0.25$ feet. Include units on your answer. What is the meaning of the value you found?

(e) More generally, what does the quantity $W_{\text{slice}} = B(h)\triangle h$ measure for a given value of h and a small positive value of $\triangle h$?

(f) Evaluate the definite integral $\int_0^{50} B(h)\,dh$. What is the meaning of the value you find? Why?

Part 2.

Consider the following situations in which a varying force accomplishes work.

(a) Suppose that a heavy rope hangs over the side of a cliff. The rope is 200 feet long and weighs 0.3 pounds per foot; initially the rope is fully extended. How much work is required to haul in the entire length of the rope? (Hint: set up a function $F(h)$ whose value is the weight of the rope remaining over the cliff after h feet have been hauled in.)

(b) A leaky bucket is being hauled up from a 100 foot deep well. When lifted from the water, the bucket and water together weigh 40 pounds. As the bucket is being hauled upward at a constant rate, the bucket leaks water at a constant rate so that it is losing weight at a rate of 0.1 pounds per foot. What function $B(h)$ tells the weight of the bucket after the bucket has been lifted h feet? What is the total amount of work accomplished in lifting the bucket to the top of the well?

(c) Now suppose that the bucket in (b) does not leak at a constant rate, but rather that its weight at a height h feet above the water is given by $B(h) = 25+15e^{-0.05h}$. What is the total work required to lift the bucket 100 feet? What is the average force exerted on the bucket on the interval $h = 0$ to $h = 100$?

(d) From physics, *Hooke's Law* for springs states that the amount of force required to hold a spring that is compressed (or extended) to a particular length is proportionate to the distance the spring is compressed (or extended) from its natural length. That is, the force to compress (or extend) a spring x units from its natural length is $F(x) = kx$ for some constant k (which is called the *spring constant.*) For springs, we choose to measure the force in pounds and the distance the spring is compressed in feet.

Suppose that a force of 5 pounds extends a particular spring 4 inches (1/3 foot) beyond its natural length.

i. Use the given fact that $F(1/3) = 5$ to find the spring constant k.

ii. Find the work done to extend the spring from its natural length to 1 foot beyond its natural length.

iii. Find the work required to extend the spring from 1 foot beyond its natural length to 1.5 feet beyond its natural length.

Part 3.

In each of the following problems, determine the total work required to accomplish the described task. In parts (b) and (c), a key step is to find a formula for a function that describes the curve that forms the side boundary of the tank.

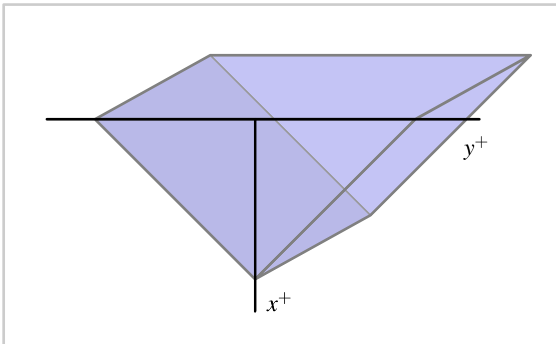

Figure: A trough with triangular ends, as described in Activity, part (c).

(a) Consider a vertical cylindrical tank of radius 2 meters and depth 6 meters. Suppose the tank is filled with 4 meters of water of mass density 1000 kg/m^3, and the top 1 meter of water is pumped over the top of the tank.

(b) Consider a hemispherical tank with a radius of 10 feet. Suppose that the tank is full to a depth of 7 feet with water of weight density 62.4 pounds/ft^3, and the top 5 feet of water are pumped out of the tank to a tanker truck whose height is 5 feet above the top of the tank.

(c) Consider a trough with triangular ends, as pictured in Figure above, where the tank is 10 feet long, the top is 5 feet wide, and the tank is 4 feet deep. Say that the trough is full to within 1 foot of the top with water of weight density 62.4 pounds/ft^3, and a pump is used to empty the tank until the water remaining in the tank is 1 foot deep.