Name________________________________

Activity - Chain Rule

Part 1. For each function given below, identify its fundamental algebraic structure. In particular, is the given function a sum, product, quotient, or composition of basic functions? If the function is a composition of basic functions, state a formula for the inner function g and the outer function f so that the overall composite function can be written in the form $f(g(x))$. If the function is a sum, product, or quotient of basic functions, use the appropriate rule to determine its derivative.

(a) $h(x) = \tan(2^x)$

(b) $p(x) = 2x\tan(x)$

(c) $r(x) = (\tan(x))^2$

(d) $m(x) = e^{\tan(x)}$

(e) $w(x) = \sqrt{x} + \tan(x)$

(f) $z(x) = \sqrt{\tan(x)}$

Part 2.

For each function given below, identify an inner function g and outer function f to write the function in the form $f(g(x))$. Then, determine $f'(x)$, $g'(x)$, and $f'(g(x))$, and finally apply the chain rule to determine the derivative of the given function.

(a) $h(x) = \cos(x^4)$

(b) $f(x) = \sin(\cos(x))$

(c) $s(x) = e^{\sin(x)}$

(d) $z(x) = \cot^5(x)$

(e) $m(x) = (\sec(x) + e^x)^9$

Part 3.

For each of the following functions, find the function's derivative. State the rule(s) you use, label relevant derivatives appropriately, and be sure to clearly identify your overall answer.

(a) $p(r) = 4\sqrt{r^6 + 2e^r}$

(b) $m(v) = \sin(v^2)\cos(v^3)$

(c) $h(y) = \dfrac{\cos(10y)}{e^{4y} + 1}$

(d) $s(z) = e^{z^2 \sec(z)}$

(e) $c(x) = \sin(e^{x^2})$

Part 4.

Use known derivative rules, including the chain rule, as needed to answer each of the following questions.

(a) Find an equation for the tangent line to the curve $y = \sqrt{e^x + 3}$ at the point where $x = 0$.

(b) If $s(t) = \dfrac{1}{(t^2+1)^3}$ represents the position function of a particle moving horizontally along an axis at time t (where s is measured in inches and t in seconds), find the particle's instantaneous velocity at $t = 1$. Is the particle moving to the left or right at that instant?

(c) At sea level, air pressure is 30 inches of mercury. At an altitude of h feet above sea level, the air pressure, P, in inches of mercury, is given by the function

$$P = 30e^{-0.0000323h}.$$

Compute dP/dh and explain what this derivative function tells you about air pressure, including a discussion of the units on dP/dh. In addition, determine how fast the air pressure is changing for a pilot of a small plane passing through an altitude of 1000 feet.

(d) Suppose that $f(x)$ and $g(x)$ are differentiable functions and that the following information about them is known:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	-5	-3	4
2	-3	4	-1	2

If $C(x)$ is a function given by the formula $f(g(x))$, determine $C'(2)$. In addition, if $D(x)$ is the function $f(f(x))$, find $D'(-1)$.