Name_______________________________________

Riemann Sums and Area Under Curves

Part 1. Go to http://webspace.ship.edu/msrenault/GeoGebraCalculus/integration_riemann_sum.html

(a) Update the applet (and view window, as needed) so that the function being considered is $f(x) = 2x + 1$ on $[1, 4]$, as directed above. For this function on this interval, compute L_n, M_n, R_n for $n = 5$, $n = 25$, and $n = 100$. What appears to be the exact area bounded by $f(x) = 2x + 1$ and the x-axis on $[1, 4]$?

(b) Use basic geometry to determine the exact area bounded by $f(x) = 2x + 1$ and the x-axis on $[1, 4]$.

(c) Based on your work in (a) and (b), what do you observe occurs when we increase the number of subintervals used in the Riemann sum?

(d) Update the applet to consider the function $f(x) = x^2 + 1$ on the interval $[1, 4]$ (note that you need to enter "`x^2 + 1`" for the function formula). Use the applet to compute L_n, M_n, R_n for $n = 5$, $n = 25$, and $n = 100$. What do you conjecture is the exact area bounded by $f(x) = x^2 + 1$ and the x-axis on $[1, 4]$?

(e) Why can we not compute the exact value of the area bounded by $f(x) = x^2 + 1$ and the x-axis on $[1, 4]$ using a formula like we did in (b)?

Part 2.

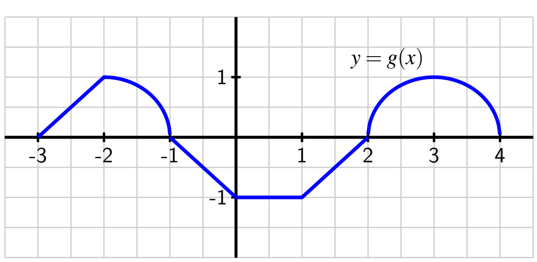
Use known geometric formulas and the net signed area interpretation of the definite integral to evaluate each of the definite integrals below.

(a) $\int_0^1 3x\,dx$

(b) $\int_{-1}^4 (2 - 2x)\,dx$

(c) $\int_{-1}^1 \sqrt{1 - x^2}\,dx$

(d) $\int_{-3}^4 g(x)\,dx$, where g is the function pictured in Figure 4.24. Assume that each portion of g is either part of a line or part of a circle.



Part 3.

The graph of g is shown. Estimate $\int_{-2}^{4} g(x)\,dx$ with six sub-intervals using (a) right endpoints, (b) left endpoints, and (c) midpoints.