Name________________________________

Activity - Substitution

Part 1. The Chain Rule can be applied to find the derivative of a composite function. In particular, if u is a differentiable function of x, and f is a differentiable function of $u(x)$, then

$$\frac{d}{dx}\left[f(u(x))\right] = f'(u(x)) \cdot u'(x).$$

In words, we say that the derivative of a composite function $c(x) = f(u(x))$, where f is considered the "outer" function and u the "inner" function, is "the derivative of the outer function, evaluated at the inner function, times the derivative of the inner function."

(a) For each of the following functions, use the Chain Rule to find the function's derivative. Be sure to label each derivative by name (e.g., the derivative of $g(x)$ should be labeled $g'(x)$).

i. $g(x) = e^{3x}$

ii. $h(x) = \sin(5x + 1)$

iii. $p(x) = \arctan(2x)$

iv. $q(x) = (2 - 7x)^4$

v. $r(x) = 3^{4-11x}$

(b) For each of the following functions, use your work in (a) to help you determine the general antiderivative[3] of the function. Label each antiderivative by name (e.g., the antiderivative of m should be called M). In addition, check your work by computing the derivative of each proposed antiderivative.

i. $m(x) = e^{3x}$

ii. $n(x) = \cos(5x + 1)$

iii. $s(x) = \frac{1}{1+4x^2}$

iv. $v(x) = (2 - 7x)^3$

v. $w(x) = 3^{4-11x}$

(c) Based on your experience in parts (a) and (b), conjecture an antiderivative for each of the following functions. Test your conjectures by computing the derivative of each proposed antiderivative.

i. $a(x) = \cos(\pi x)$

ii. $b(x) = (4x + 7)^{11}$

iii. $c(x) = xe^{x^2}$

Part 2.

Evaluate each of the following indefinite integrals. Check each antiderivative that you find by differentiating.

(a) $\int \sin(8-3x)\,dx$

(b) $\int \sec^2(4x)\,dx$

(c) $\int \frac{1}{11x-9}\,dx$

(d) $\int \csc(2x+1)\cot(2x+1)\,dx$

(e) $\int \frac{1}{\sqrt{1-16x^2}}\,dx$

(f) $\int 5^{-x}\,dx$

Part 3.

Evaluate each of the following indefinite integrals by using these steps:

- Find two functions within the integrand that form (up to a possible missing constant) a function-derivative pair;
- Make a substitution and convert the integral to one involving u and du;
- Evaluate the new integral in u;
- Convert the resulting function of u back to a function of x by using your earlier substitution;
- Check your work by differentiating the function of x. You should come up with the integrand originally given.

(a) $$\int \frac{x^2}{5x^3+1}\,dx$$

(b) $$\int e^x \sin(e^x)\,dx$$

(c) $$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$$

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Part 4.

Evaluate each of the following definite integrals exactly through an appropriate u-substitution.

(a) $\displaystyle\int_1^2 \frac{x}{1+4x^2}\,dx$

(b) $\displaystyle\int_0^1 e^{-x}(2e^{-x}+3)^9\,dx$

(c) $\displaystyle\int_{2/\pi}^{4/\pi} \frac{\cos\left(\frac{1}{x}\right)}{x^2}\,dx$