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Activity - Integration by Parts Prep

Part 1

In Calculus I you learned the Product Rule and studied how it is employed to differentiate a product of two functions. In particular, recall that if f and g are differentiable functions of x, then

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x).$$

(a) For each of the following functions, use the Product Rule to find the function's derivative. Be sure to label each derivative by name (e.g., the derivative of $g(x)$ should be labeled $g'(x)$).

i. $g(x) = x\sin(x)$

ii. $h(x) = xe^x$

iii. $p(x) = x\ln(x)$

iv. $q(x) = x^2\cos(x)$

v. $r(x) = e^x\sin(x)$

(b) Use your work in (a) to help you evaluate the following indefinite integrals. Use differentiation to check your work.

i. $\displaystyle\int xe^x + e^x\,dx$

ii. $\displaystyle\int e^x(\sin(x) + \cos(x))\,dx$

iii. $\displaystyle\int 2x\cos(x) - x^2\sin(x)\,dx$

iv. $\displaystyle\int x\cos(x) + \sin(x)\,dx$

v. $\displaystyle\int 1 + \ln(x)\,dx$

(c) Observe that the examples in (b) work nicely because of the derivatives you were asked to calculate in (a). Each integrand in (b) is precisely the result of differentiating one of the products of basic functions found in (a). To see what happens when an integrand is still a product but not necessarily the result of differentiating an elementary product, we consider how to evaluate

$$\int x \cos(x)\, dx.$$

i. First, observe that

$$\frac{d}{dx}\left[x \sin(x)\right] = x \cos(x) + \sin(x).$$

Integrating both sides indefinitely and using the fact that the integral of a sum is the sum of the integrals, we find that

$$\int \left(\frac{d}{dx}\left[x \sin(x)\right]\right) dx = \int x \cos(x)\, dx + \int \sin(x)\, dx.$$

In this last equation, evaluate the indefinite integral on the left side as well as the rightmost indefinite integral on the right.

ii. In the most recent equation from (i.), solve the equation for the expression $\int x \cos(x)\, dx$.

iii. For which product of basic functions have you now found the antiderivative?

Part 2

Evaluate each of the following indefinite integrals. Check each antiderivative that you find by differentiating.

(a) $\int te^{-t}\, dt$

(b) $\int 4x \sin(3x)\, dx$

(c) $\int z \sec^2(z)\, dz$

(d) $\int x \ln(x)\, dx$

Part 3.

Evaluate each of the following indefinite integrals, using the provided hints.

(a) Evaluate $\int \arctan(x)\,dx$ by using Integration by Parts with the substitution $u = \arctan(x)$ and $dv = 1\,dx$.

(b) Evaluate $\int \ln(z)\,dz$. Consider a similar substitution to the one in (a).

(c) Use the substitution $z = t^2$ to transform the integral $\int t^3 \sin(t^2)\,dt$ to a new integral in the variable z, and evaluate that new integral by parts.

(d) Evaluate $\int s^5 e^{s^3}\,ds$ using an approach similar to that described in (c).

(e) Evaluate $\int e^{2t}\cos(e^t)\,dt$. You will find it helpful to note that $e^{2t} = e^t \cdot e^t$.

Part 4.

Evaluate each of the following indefinite integrals.

(a) $\displaystyle\int x^2 \sin(x)\,dx$

(b) $\displaystyle\int t^3 \ln(t)\,dt$

(c) $\displaystyle\int e^z \sin(z)\,dz$

(d) $\displaystyle\int s^2 e^{3s}\,ds$

(e) $\displaystyle\int t\arctan(t)\,dt$
(**Hint**: At a certain point in this problem, it is very helpful to note that $\frac{t^2}{1+t^2} = 1 - \frac{1}{1+t^2}$.)

Part 5. For each of the indefinite integrals below, the main question is to decide whether the integral can be evaluated using u-substitution, integration by parts, a combination of the two, or neither. For integrals for which your answer is affirmative, state the substitution(s) you would use. It is not necessary to actually evaluate any of the integrals completely, unless the integral can be evaluated immediately using a familiar basic antiderivative.

(a) $\int x^2 \sin(x^3)\,dx, \quad \int x^2 \sin(x)\,dx, \quad \int \sin(x^3)\,dx, \quad \int x^5 \sin(x^3)\,dx$

(b) $\int \frac{1}{1+x^2}\,dx, \quad \int \frac{x}{1+x^2}\,dx, \quad \int \frac{2x+3}{1+x^2}\,dx, \quad \int \frac{e^x}{1+(e^x)^2}\,dx,$

(c) $\int x\ln(x)\,dx, \quad \int \frac{\ln(x)}{x}\,dx, \quad \int \ln(1+x^2)\,dx, \quad \int x\ln(1+x^2)\,dx,$

(d) $\int x\sqrt{1-x^2}\,dx, \quad \int \frac{1}{\sqrt{1-x^2}}\,dx, \quad \int \frac{x}{\sqrt{1-x^2}}\,dx, \quad \int \frac{1}{x\sqrt{1-x^2}}\,dx,$