Name________________________________

Activity - Product and Quotient Rules

Part 1. Let f and g be the functions defined by $f(t) = 2t^2$ and $g(t) = t^3+4t$.

(a) Determine $f'(t)$ and $g'(t)$.

(b) Let $p(t) = 2t^2(t^3 + 4t)$ and observe that $p(t) = f(t) \cdot g(t)$. Rewrite the formula for p by distributing the $2t^2$ term. Then, compute $p'(t)$ using the sum and constant multiple rules.

(c) True or false: $p'(t) = f'(t) \cdot g'(t)$.

(d) Let $q(t) = \dfrac{t^3 + 4t}{2t^2}$ and observe that $q(t) = \dfrac{g(t)}{f(t)}$. Rewrite the formula for q by dividing each term in the numerator by the denominator and simplify to write q as a sum of constant multiples of powers of t. Then, compute $q'(t)$ using the sum and constant multiple rules.

(e) True or false: $q'(t) = \dfrac{g'(t)}{f'(t)}$.

Part 2. Find the derivative of $P(x) = f(x) \cdot g(x)$ by doing the following: (we assume that $f(x)$ and $g(x)$ are differentiable functions).

a) Find $P'(x)$ by applying the limit definition to $f(x) \cdot g(x)$ to get an expression for the derivative in terms of the functions f and g and variables x and h.

b) In between the two terms in the numerator, subtract and then add the expression $f(x)g(x+h)$. The reason we do this will become apparent as we continue. Does this leave an equivalent expression when we do this?

c) Split the resulting expression into two fractions, one using the first 2 terms of the numerator and the second using the last two terms. (Each fraction will have the same denominator).

d) Apply limit properties to change this limit of sums into the sum of the limits.

e) In each limit expression, factor the common factor out in front of the fraction.

f) Apply limit properties to change the limit of the products into the products of the limits in these expressions.

g) Do you recognize two of these limits?

Replace these limits with an equivalent derivative and simplify.

Complete the Product Rule below.

Product Rule: If *f* and *g* are differentiable functions, then their product $P(x) = f(x) \cdot g(x)$ is also a differentiable function, and

$$P'(x) =$$

Part 1.

Let $Q(x)$ be defined by $Q(x) = f(x)/g(x)$, where f and g are both differentiable functions. We desire a formula for Q' in terms of f, g, f', and g'. It turns out that Q is differentiable everywhere that $g(x) \neq 0$.

(a) Take the formula $Q = f/g$ and multiply both sides by g.

(b) Use the product rule to differentiate f.

(c) Solve the resulting equation for $Q'(x)$.

(d) Divide both sides by $g(x)$.

(e) Recall that $Q(x) =$ f&x'-g&x', Sse this expression in the preceding equation to replace $Q(x)$.

(f) Simply the complex fraction and express as a single fraction.

Complete the Quotient Rule below.

Quotient Rule: If f and g are differentiable functions, then their quotient $Q(x) = \frac{f(x)}{g(x)}$ is also a differentiable function for all x where $g(x) \neq 0$, and

$$Q'(x) = \underline{\qquad\qquad\qquad}.$$