Name________________________________

Activity - Continuity

On a separate paper, label the parts and answer the following questions. When asked to plot the function, do so on the graph provided. Attach your answer paper to this assignment.

Part 1. A function f defined on $-4 < x < 4$ is given by the graph in Figure below. Use the graph to answer each of the following questions. Note: to the right of $x = 2$, the graph of f is exhibiting infinite oscillatory behavior.

For each part, make a table with values of a in the first column and then columns for each of the other things you are asked to do for that part. Attach the tables to this assignment.

(a) For each of the values $a = -3, -2, -1, 0, 1, 2, 3$, determine whether or not $\lim_{x\to a} f(x)$ exists. If the function has a limit L at a given point, state the value of the limit using the notation $\lim_{x\to a} f(x) = L$. If the function does not have a limit at a given point, write a sentence to explain why.

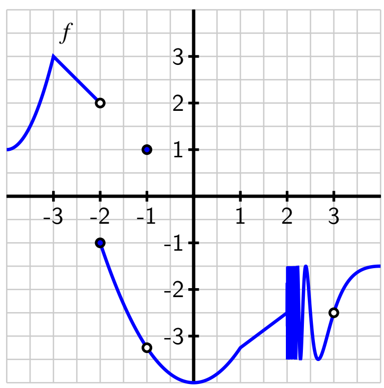

Figure: The graph of $y = f(x)$.

(b) For each of the values of a from part (a) where f has a limit, determine the value of $f(a)$ at each such point. In addition, for each such a value, does $f(a)$ have the same value as $\lim_{x\to a} f(x)$?

(c) For each of the values $a = -3, -2, -1, 0, 1, 2, 3$, determine whether or not $f'(a)$ exists. In particular, based on the given graph, ask yourself if it is reasonable to say that f has a tangent line at $(a, f(a))$ for each of the given a-values. If so, visually estimate the slope of the tangent line to find the value of $f'(a)$.

Part 2.

Consider a function that is piecewise-defined according to the formula

$$f(x) = \begin{cases} 3(x+2)+2 & \text{for } -3 < x < -2 \\ \frac{2}{3}(x+2)+1 & \text{for } -2 \le x < -1 \\ \frac{2}{3}(x+2)+1 & \text{for } -1 < x < 1 \\ 2 & \text{for } x = 1 \\ 4 - x & \text{for } x > 1 \end{cases}$$

Make a table of the values below and use the given formula to answer the following questions as columns in the table. Attach the table to this assignment.

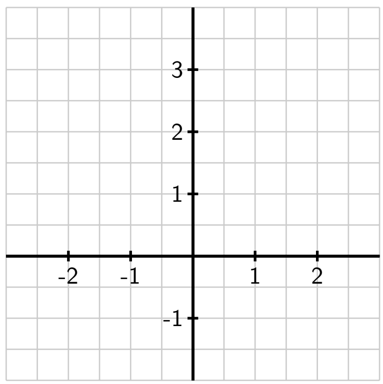

Figure: Axes for plotting the function $y = f(x)$

(a) For each of the values $a = -2, -1, 0, 1, 2$, compute $f(a)$.

(b) For each of the values $a = -2, -1, 0, 1, 2$, determine $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$.

(c) For each of the values $a = -2, -1, 0, 1, 2$, determine $\lim_{x \to a} f(x)$. If the limit fails to exist, explain why by discussing the left- and right-hand limits at the relevant a-value.

(d) For which values of a is the following statement true?

$$\lim_{x \to a} f(x) \neq f(a)$$

(e) On the axes provided above, sketch an accurate, labeled graph of $y = f(x)$. Be sure to carefully use open circles (○) and filled circles (•) to represent key points on the graph, as dictated by the piecewise formula.

Part 3.

This activity builds on your work in Part 1, using the same function f as given by the graph in Part 1.

(a) At which values of a does $\lim_{x\to a} f(x)$ not exist?

(b) At which values of a is $f(a)$ not defined?

(c) At which values of a does f have a limit, but $\lim_{x\to a} f(x) \neq f(a)$?

(d) State all values of a for which f is not continuous at $x = a$.

(e) Which condition is stronger, and hence implies the other: f has a limit at $x = a$ or f is continuous at $x = a$? Explain, and hence complete the following sentence: "If f ______________________ at $x = a$, then f ______________________ at $x = a$," where you complete the blanks with ***has a limit*** and ***is continuous***, using each phrase once.