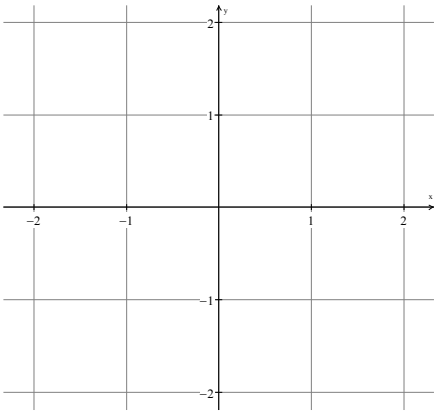
Activity – Hyperbolic Functions Name_____

Part 1 – Basics of Hyperbolic Functions

Recall from trigonometry the parametric equations $x = \cos(t)$ and $y = \sin(t)$.

Graph these equations.



Some combinations of exponential functions occur frequently in applications and have been given the special names of hyperbolic functions. In particular, we are going to consider hyperbolic cosines and hyperbolic sines which are abbreviated cosh(*t*) and sinh(*t*). Notice they look a lot like cos(t) and sin(t), but in fact, they are not trig functions at all, but as we will see, they have some similar properties and hence contain sines or cosines in their names. But, what about the hyperbolic part of their name?

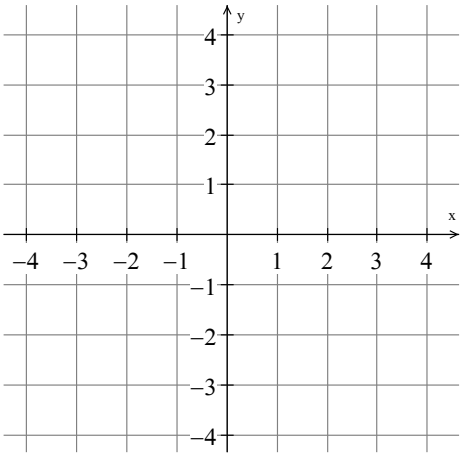
First, we need the definitions of these functions. Complete the definitions below:

$\cosh t =$

$\sinh t =$

Let's graph the parametric equations $x(t) = \cosh(t)$ and $y(t) = \sinh(t)$.

t	cosh(*t*)	sinh(*t*)
−2		
−1		
0		
1		
2		



Now thinking back to conic sections in college algebra, graph $\frac{x^2}{1} - \frac{y^2}{1} = 1$ on the same graph. What do you discover?

Part 2. Calculus and Hyperbolic Functions

Find the derivatives of $\cosh(x)$ and $\sinh(x)$ by using their definitions and differentiating exponentials. Give final answers in terms of hyperbolic functions.

Part 3. Looking for Analogies

a) Looking at the derivatives above, how are the derivatives of the hyperbolic cosines and hyperbolic sines similar to the derivatives for cosines and sines?

b) For $x = 0$, find the values of $\cos(x), \sin(x), \cosh(x),$ and $\sinh(x)$. Write a sentence about any analogies you see between the trig functions and the hyperbolic functions.

c) Using their definitions, find $\cosh^2(x) - \sinh^2(x)$. Is this similar to anything you've seen using sines and cosines?

d) Using the definition, find $\sinh(2x)$. Factor the numerator. Can you find a way to express this using a hyperbolic sine and hyperbolic cosine? Any analogies to trig functions here?

e) Using the definition, find $\cosh(-x)$ and $\sinh(-x)$. Express your answers back in terms of hyperbolic functions. Analogies to trig functions?