Name____

Activity – Vectors

Part 1. Vector Notation and Terminology

A vector from point P to point Q is written $\overrightarrow{PQ}$.

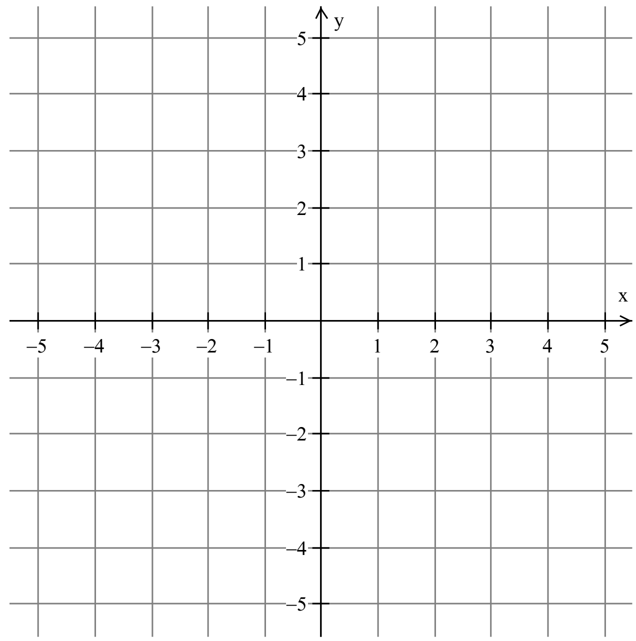
a) Plot the vector determined by $P(-3,1)$ and $Q(2,4)$.

b) Find the length of $\overrightarrow{PQ}$.

The length of a vector is called its **magnitude and it denoted** $\left\|\overrightarrow{PQ}\right\|$ **.**

c) If you picked up the vector and moved it so that its tail (or initial point) was at the origin, at what point would its tip or head be? Draw this vector. This vector is considered equal to your original vector.

Can you see how to get this point from your original points $P(-3,1)$ and $Q(2,4)$?

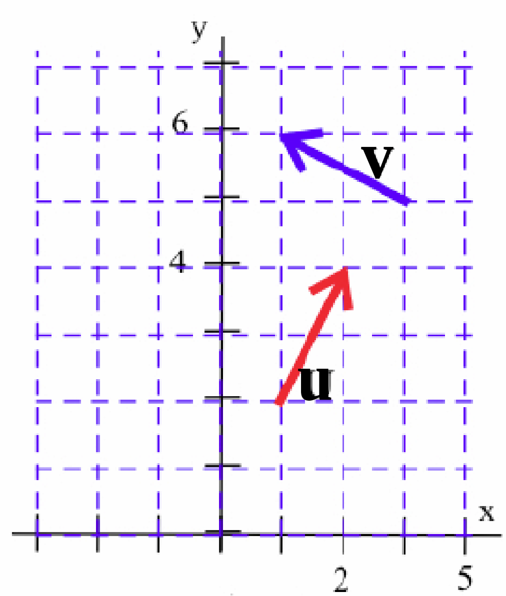


A vector with its tail at the origin is called a vector in standard position and is often denoted **v** or $\vec{v}$.

d) Since the tail is at the origin we can express the vector by the single point (a, b) that is its tip . Vector notation for this is $\langle a,b\rangle$ and it is called **component form** . When in these type brackets, we know it is a vector in standard position instead of a point. Express the vector $\overrightarrow{PQ}$ with this notation.

e) As well as a magnitude, vectors have a **direction**. The directionangle θ is an angle in standard position whose terminal side contains the vector. Direction angles of vectors are typically in degrees with $0° \le \theta < 360°$ though they may be in radians or with coterminal angles . Find the direction angle of vector $\overrightarrow{PQ}$ above.

f) Using the graph at right, express vectors **u** and **v** in component form.



g) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

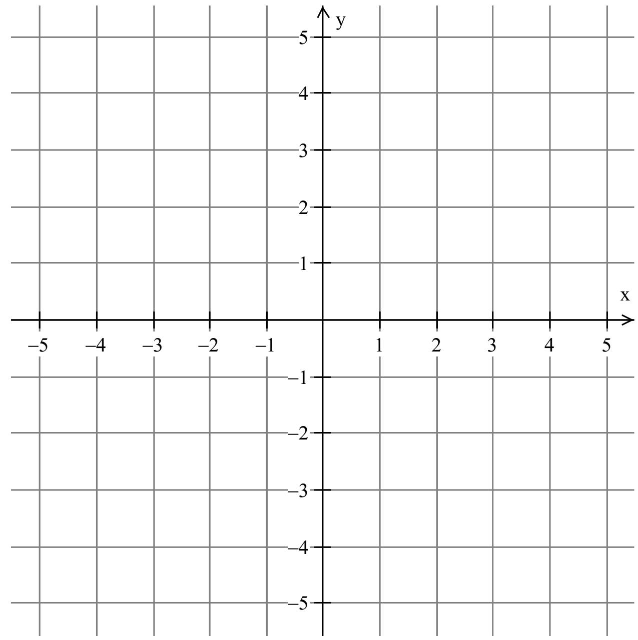
h) Find the direction angles of **u** and **v** respectively.

i) Can you find a vector that is in the direction of **v** but is only one unit long? This is called a **unit vector** in the direction of **v** .

Part 2. Vector Addition and Scalar Multiplication

1. To add vectors together geometrically, you put the tail of the second one to the tip of the first and the resultant vector is the vector that starts where you started and ends where you ended. To see this, let's add $\mathbf{v}+\mathbf{u}$ where $\mathbf{v}=\langle 2,1\rangle$ and $\mathbf{u}=\langle 1,3\rangle$ by doing the following:

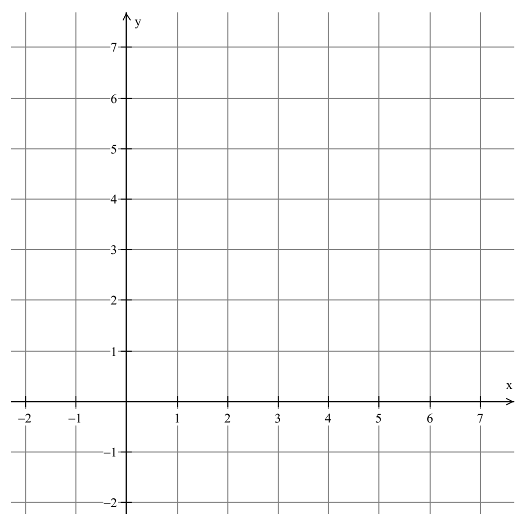
   a) Plot $\mathbf{v}$ and $\mathbf{u}$ in standard position.
   b) Now slide $\mathbf{u}$ so that its tail is on the tip of $\mathbf{v}$.
   c) Now draw that resultant vector that runs from the tail of **v** to the tip of **u**.
   d) Express this vector $\mathbf{v}+\mathbf{u}$ in component form.



   e) Look at the components of $\mathbf{v}$ and $\mathbf{u}$. Can you see how to add two vectors in component form without plotting them?

2. A number multiplied in front of a vector is called a **scalar.** It means to take the vector and add it together that many times.

   a) Plot $\mathbf{v}=\langle 2,1\rangle$.
   b) Now plot 3**v** by adding **v** together 3 times.
   c) What do you suppose –1**v = –v** would mean?



   d) If $\mathbf{u}=\langle 1,3\rangle$, plot $\mathbf{u}-\mathbf{v}=\mathbf{u}+(-\mathbf{v})$ and express the resultant vector in component form.
   f) Look at the components of $\mathbf{v}$ and $\mathbf{u}$. Can you see how to subtract two vectors in component form without plotting them?

   e) Without plotting, find $5\mathbf{u}-3\mathbf{v}$.

Thinking of what you know about how to add vectors, fill in the following:

Properties of Vector Addition

- **Commutative Property:** For all vectors $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v}+\mathbf{w}=$ _________
- **Associative Property:** For all vectors $\mathbf{u},\mathbf{v}$ and $\mathbf{w}$, $(\mathbf{u}+\mathbf{v})+\mathbf{w}=$ ______________________
- **Additive Identity Property:** For all vectors $\mathbf{v}, \mathbf{v}+\mathbf{0}=$ __________
- **Inverse Property:** Every vector $\mathbf{v}$ has a unique additive inverse denoted $-\mathbf{v}$ so $\mathbf{v}+(-\mathbf{v})=$ ____

Thinking of what you know about scalar multiplication, fill in the following:

Properties of Scalar Multiplication

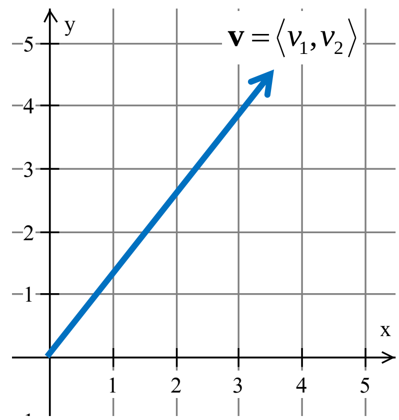
If k is a real number and $\mathbf{v}=\langle v_1, v_2\rangle$, we define $k\mathbf{v}=k\langle v_1, v_2\rangle=$ _____________________

- **Associative Property:** For all vectors **v** and scalars *k* and *r*, $(kr)\mathbf{v}=$ _______
- **Multiplicative Identity Property:** For all vectors $\mathbf{v}, 1\mathbf{v}=$ ____
- **Additive Inverse Property:** For all vectors **v**, $-\mathbf{v}=$ __________
- **Distributive Property of Scalar Multiplication over Scalar Addition:** For all vectors **v** and scalars *k* and *r*, $(k+r)\mathbf{v}=$ ______________
- **Distributive Property of Scalar Multiplication over Vector Addition:** For all vectors **v** and **w** and scalars *k*, $k(\mathbf{v}+\mathbf{w})=$ ___________
- **Zero Product Property:** If **v** is a vector and *k* is a scalar, then $k\mathbf{v}=\mathbf{0}$ if and only if ____= 0 or ____= **0**.

Part 3. Magnitude and Direction of a Vector

Remember all vectors can be defined by their magnitude and direction. They can be given in component form or in polar (or trigonometric) form.

1. Given the vector $\mathbf{v} = \langle -2, -5 \rangle$, find its magnitude and direction angle.



2. a) For the vector shown, find $\|\mathbf{v}\|$.

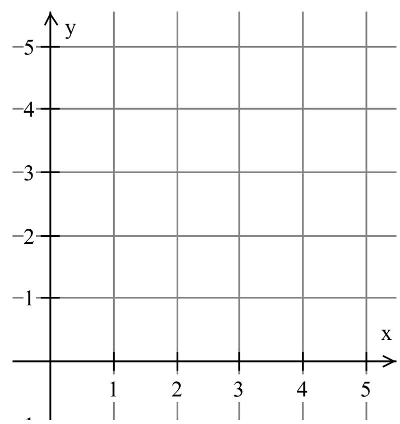
   b) Make a right triangle with **v** as the hypotenuse and label the lengths of the three sides and put in the direction angle θ.

   c) Find v_1 and v_2 in terms of the sides and the direction angle and use these to express $\mathbf{v} = \langle v_1, v_2 \rangle$.

This is called the polar or trigonometric form of the vector. The $\|\mathbf{v}\|$ is often factored out.

3. Given a vector **v** has a magnitude of 4 and a direction angle of 300°, express it in component form.

Part 4. Expressing Vectors in i, j Form.



1. a) Plot vector $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$. What is $\|\mathbf{i}\|$? $\|\mathbf{j}\|$?

b) These are called **principal unit vectors** and any vector can be expressed by a linear combination (scalar multiplying and adding) of these two vectors. Plot the vector $\mathbf{v} = \langle 2, 4 \rangle$. Now use $k_1\mathbf{i} + k_2\mathbf{j}$ to express **v** and show this vector scalar multiplication and addition on your plot.

Wrap up. Given vectors $\mathbf{v} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$ and $\mathbf{w} = \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle$ **,** first state whether the quantity is a scalar or a vector and then find the quantity.

$a)\ \mathbf{v} + \mathbf{w}$ $\quad b)\ \mathbf{w} - 2\mathbf{v}$ $\quad c)\ \|\mathbf{v} + \mathbf{w}\|$ $\quad d)\ \|\mathbf{v}\| + \|\mathbf{w}\|$ $\quad e)\ \|\mathbf{w}\|\mathbf{v}$ $\quad f)\ \frac{\mathbf{v}}{\|\mathbf{v}\|}$