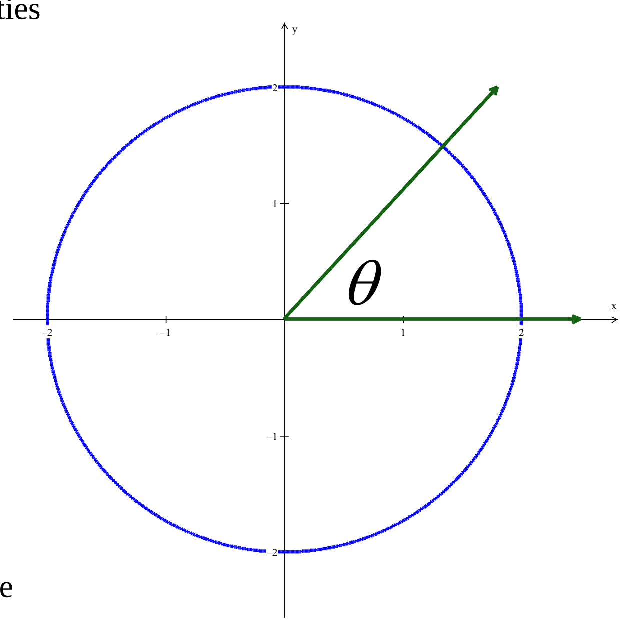
Name____

Activity – Deriving Identities



Part 1. The Even/Odd Identities

Angle θ is shown in standard position in the circle below.

a) Draw the angle $(-\theta)$.

b) Consider the values of $\cos\theta$ and $\cos(-\theta)$.
Would these values be the same or different? If different, how are they different?

c) Consider the values of $\sin\theta$ and $\sin(-\theta)$. Would these values be the same or different? If different, how are they different?

d) Consider the values of $\tan\theta$ and $\tan(-\theta)$. Would these values be the same or different? If different, how are they different?

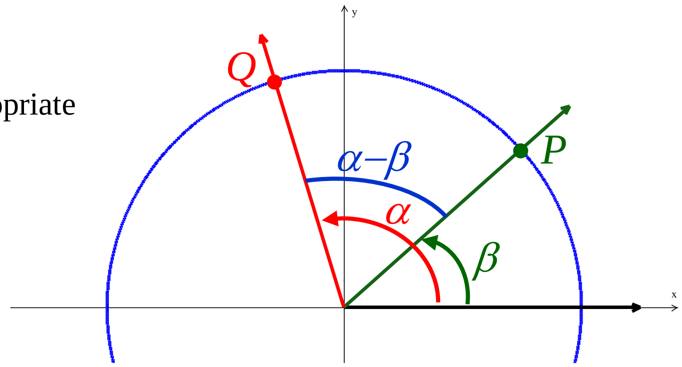
e) Draw an angle in standard position whose terminal side is in Quadrant II and label it α. Consider the sine, cosine and tangent of α and of $(-\alpha)$. Would the things you saw about the connection between the values for trig functions of positive and negative angles in parts b, c and d hold for an angle in Quadrant II? What about Quadrant III? Quadrant IV?

Complete the table:

$\cos(-\theta) =$	$\sin(-\theta) =$	$\tan(-\theta) =$
$\sec(-\theta) =$	$\csc(-\theta) =$	$\cot(-\theta) =$

Part 2. The Sum and Difference Identities for Cosine

The circles shown are **unit** circles.

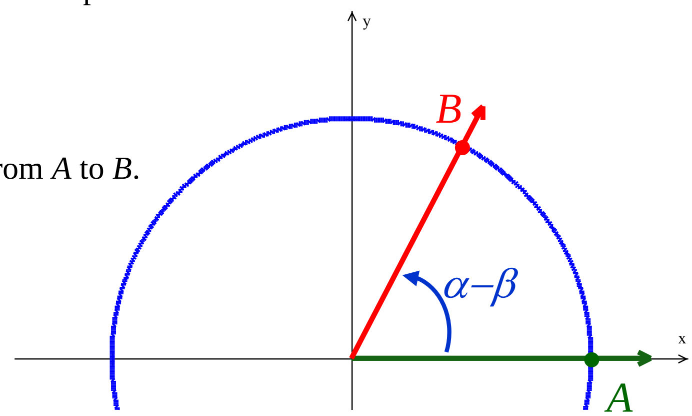


a) Label the (x, y) coordinates of points P and Q with the appropriate trig functions.

b) Draw a triangle by joining points P and Q.

c) Use the distance formula to find the length of the side of the triangle you just drew.

d) We want to take the angle $\alpha-\beta$ and place it in standard position. This is shown in the circle below. Label the (x, y) coordinates for points A and B on this circle.



e) Draw a triangle by joining points A and B. This triangle is congruent to the triangle you found in the top circle.

f) Use the distance formula to find the length of the segment from A to B.

g) Since the two lengths you found are corresponding sides of congruent triangles, they are equal. Set the two equal to each other and simplify.

h) Solve for $\cos(\alpha-\beta)$.

i) This is called the difference identity for cosine. Fill it in on your identity reference sheet.

j) $\cos(\alpha+\beta) = \cos(\alpha-(-\beta))$. Use the difference identity for cosine and use the even/odd identities to derive the sum identity for cosine and add it to your reference sheet. (Hint: So this will involve subbing in $(-\beta)$ for β.)

Part 3. Cofunction Identities & Sum and Difference Identities for Sine

a) Apply the difference identity for cosine to $\cos\left(\frac{\pi}{2}-\theta\right)$ and simplify. This is called a Cofunction Identity.

b) Since $\sin(\theta)=\cos\left(\frac{\pi}{2}-\theta\right)$, we can find $\sin\left(\frac{\pi}{2}-\theta\right)$ by taking the angles of the trig functions (called the arguments) and subtracting them from $\frac{\pi}{2}$. $\sin\left(\frac{\pi}{2}-(\theta)\right)=\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{2}-\theta\right)\right)$.

Simplify to get another cofunction identity.

c) We are now going to use identities we know to derive a sum identity for sine, namely $\sin(\alpha+\beta)$. Use the appropriate cofunction identity to re-write this in terms of cosine. Hint: $\theta=\alpha+\beta$.

We can rewrite this as: $\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\frac{\pi}{2}-\alpha-\beta\right)=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)$.

Apply the difference identity for cosine to this last expression and simplify using cofunction identities.

Remember that we started out with $\sin(\alpha+\beta)$ so you final expression is equal to this and the two form the Sum Identity for Sine. You can add this to your identity reference sheet.

d) Find a Difference Identity for Sine. (Hint: Rewrite $\sin(\alpha-\beta)$ as $\sin(\alpha+(-\beta))$ and use the Sum Identity for Sine.

e) Derive a cofunction identity for $\tan\left(\frac{\pi}{2}-\theta\right)$ by using the difference identities for sine and cosine.

f) Find a cofunction identity for $\cot\left(\frac{\pi}{2}-\theta\right)$ in the same way we found one for $\sin\left(\frac{\pi}{2}-\theta\right)$.

g) Can you find a cofunction identity for $\sec\left(\frac{\pi}{2}-\theta\right)$?

h) Can you find a cofunction identity for $\csc\left(\frac{\pi}{2}-\theta\right)$?

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Part 4. Double Angle Identities

a) The expression $\sin(2\theta)$ is called a double angle because θ is doubled. Often we want to express it using just θ (not doubled), so we will derive a double angle formula for sine by expressing $\sin(2\theta)$ as $\sin(\theta+\theta)$. Use the Sum Identity for Sine to express this in terms of just θ.

b) Do the same thing for $\cos(2\theta)$.

c) You can use Pythagorean Identities to express this identity in terms of either just $\cos^2\theta$ or just $\sin^2\theta$. Do each of these to get two more forms of the Double Angle Identity for Cosine.

d) Use your results from parts a and b to find $\tan(2\theta)$. Hint: After expressing tangent in terms of sine and cosine, divide numerator and denominator by $\cos^2\theta$ and simplify, expressing in terms of tangent.

Part 5. Power Reducing and Half-Angle Identities

a) Take one of your alternate forms for $\cos(2\theta)$ and solve it for the squared trig function.

This is a useful formula to express a squared trig function with one that is not squared. It will be **very** useful if you learn calculus.

b) Now take the other alternate form and solve for the other squared trig function. These two identities go on your identity sheet under Power Reduction Identities.

c) To derive a Half Angle Identity, take the first Power Reduction Identity and use the substitution $\theta = \frac{\alpha}{2}$.

Now take the square root of both sides and you have the first half angle identity.

d) Repeat the process in part c using the other Power Reduction Identity. Add these new identities to your Identity Reference Sheet.

e) Find a half-angle identity for tangent.