

Activity - Optimization

Part 1.

According to U.S. postal regulations, the girth plus the length of a parcel sent by mail may not exceed 108 inches, where by “girth” we mean the perimeter of the smallest end. What is the largest possible volume of a rectangular parcel with a square end that can be sent by mail? What are the dimensions of the package of largest volume?

- (a) Let x represent the length of one side of the square end and y the length of the longer side. Label these quantities appropriately on the image shown.

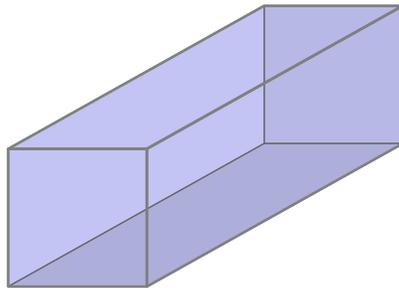


Figure: A rectangular parcel with a square end.

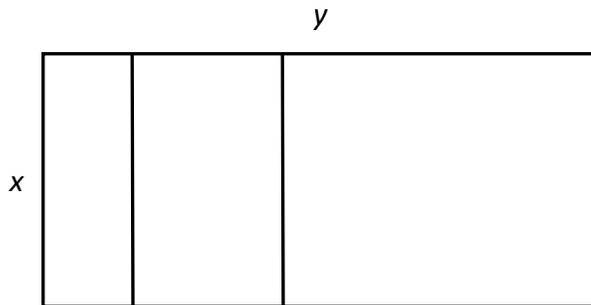
- (b) What is the quantity to be optimized in this problem? Find a formula for this quantity in terms of x and y .
- (c) The problem statement tells us that the parcel’s girth plus length may not exceed 108 inches. In order to maximize volume, we assume that we will actually need the girth plus length to equal 108 inches. What equation does this produce involving x and y ?
- (d) Solve the equation you found in (c) for one of x or y (whichever is easier).
- (e) Now use your work in (b) and (d) to determine a formula for the volume of the parcel so that this formula is a function of a single variable.
- (f) Over what domain should we consider this function? Note that both x and y must be positive; how does the constraint that girth plus length is 108 inches produce intervals of possible values for x and y ?
- (g) Find the absolute maximum of the volume of the parcel on the domain you established in (f) and hence also determine the dimensions of the box of greatest volume. Justify that you’ve found the maximum using calculus.

Part 2.

A box with a square base and open top must have a volume of 1600cm^3 . Find the dimensions of the box that minimize the amount of material used.

Part 3.

A rancher wants to fence in a large rectangular area with the overall outer dimensions x and y . She then wants to partition the area into three rectangular pens for various animals as shown below (note that the three pens are not necessarily the same size).



If exactly 800 m of fencing material is to be used, what is the maximum total enclosed area?

Part 4.

A rectangular sheet of perimeter 36 cm and dimensions x cm by y cm is to be rolled into a cylinder as shown in part of the figure. What values of x and y give the largest volume?

Useful formulas: area A of a circle, circumference C of a circle, volume V of a cylinder where r = radius, h = height.

$$A = \pi r^2; \quad C = 2\pi r; \quad V = \pi r^2 h$$

