Math 1010 Lab Activity: Quadratic Functions and Their Graphs

Name: ________________________________

A quadratic function is a second degree polynomial function. There are two common forms of the equation:

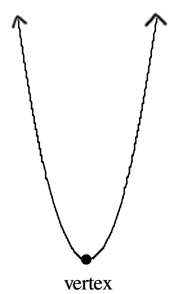
standard form: $f(x) = ax^2 + bx + c$

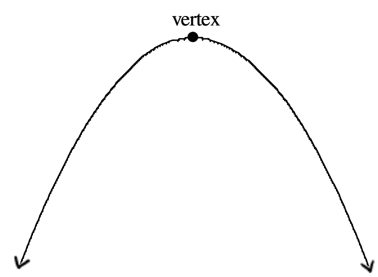
(h, k) form: $f(x) = a(x - h)^2 + k$

This lab will focus on the (h, k) form and how to use this form to easily graph the function.

PART I: Graphing Basics

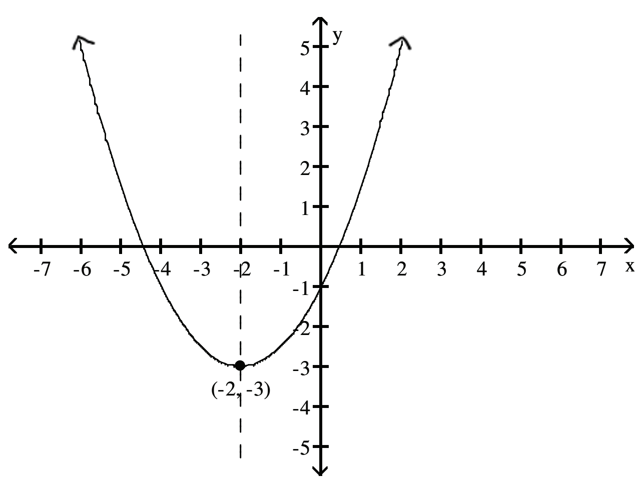
The graph of a quadratic function will always be a parabola with this basic shape





A key point of interest is the vertex, which is the lowest or minimum point on the graph of a parabola that opens up, or the highest or maximum point on the graph of a parabola that opens down. Another important property of these graphs is that they are symmetric about the vertical line that passes through the vertex so that each side is a mirror image of the other. This vertical line is called the axis of symmetry.

Example:



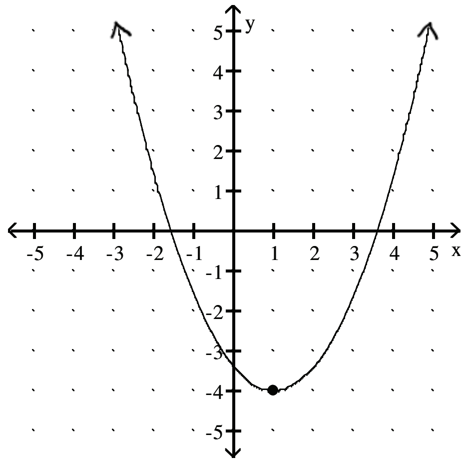
In this graph the vertex is the point $(-2, -3)$.

The equation of the axis of symmetry is $x = -2$.

Note: the axis of symmetry is a line. It can only be correctly described by an equation.

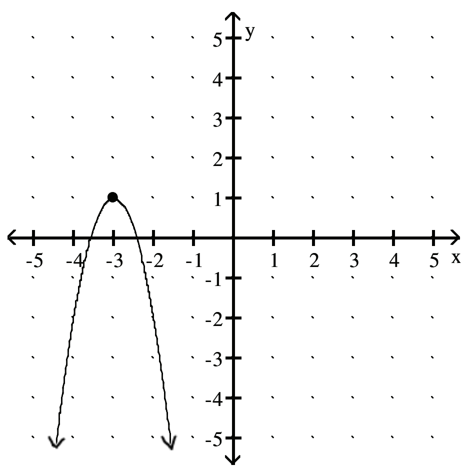
For each graph shown below, identify the vertex and the equation of the axis of symmetry.

1.



vertex ________________ equation of the axis of symmetry __________________

2.



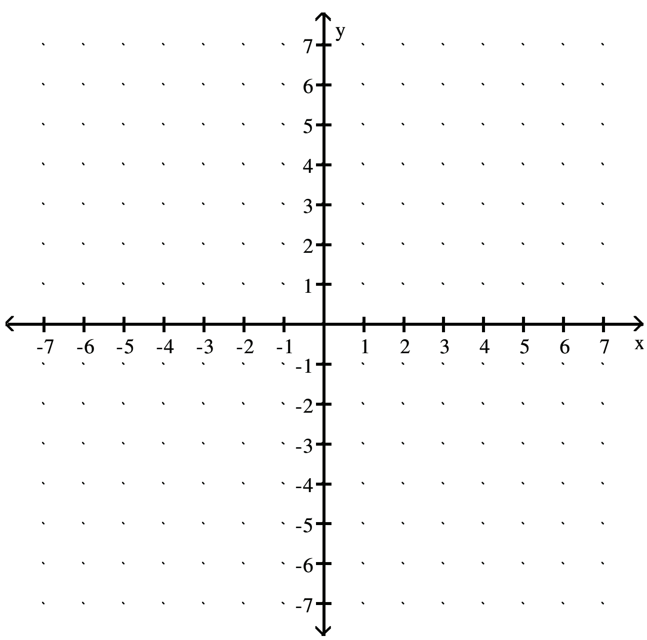
vertex ________________ equation of the axis of symmetry __________________

PART II: Graphing $f(x) = x^2$

The easiest parabola to graph, and the one to which we will compare others, is the function $f(x) = x^2$ or $y = x^2$ which you will graph by plotting points.

1. State the domain of this function.

2. Find at least five exact (x, y) ordered pairs that are on the graph and sketch the graph of the function.



What point is the vertex of this graph?

What is the equation of the axis of symmetry for this graph?

PART III: Graphing Transformations

Now you will experiment with the graph in the form of $f(x) = a(x - h)^2 + k$. Since graphing by hand is slow and tedious, you will be using online graphing software. Note that your correct placement of parenthesis is important in this process. To raise an expression to a power type ^ which is the shift of the 6 key. For example to enter $(x + 3)^2$, type (x + 3)^2.

Graphing software is located at: http://www.meta-calculator.com/online/

First click on "graphing calculator" and in equation 1, graph $y = x^2$ by typing x^2 after the "y =" prompt. Now click on "graph" and you will see your parabola graphed in red.

Leave the graph of $y = x^2$ in equation 1 for the whole exercise so that you may compare all the other graphs to this basic function.

Graphing $y = x^2 + k$

1. In equation 2, graph $y = x^2 + 2$. Click on the graph button and you will see your original $y = x^2$ in red and your new graph $y = x^2 + 2$ on the same axes graphed in blue.

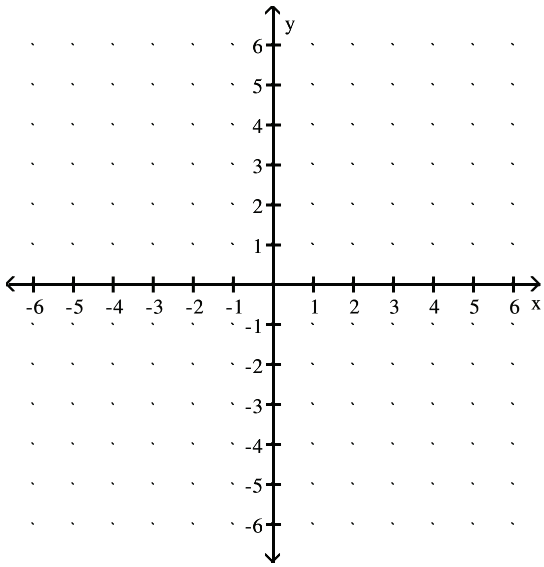
   What happened? Did adding 2 cause the vertex to move to a different location? If so where is the vertex of $y = x^2 + 2$?

2. Click on the equation button and in equation 3, type $y = x^2 - 5$.

   See if you and your lab buddy can predict what this graph will look like before clicking on the graph button. Were you correct?

   What is the vertex of the graph $y = x^2 - 5$?

3. Sketch both equations, $y = x^2 + 2$ and $y = x^2 - 5$ on the plane below. Label which graph is which and label the vertex on each graph.



Try graphing $y = x^2 + k$ with various values for k until you are confident that you know what the graph will look like.

<u>Generalize the concept:</u>

How does the graph of $y = x^2 + k$ compare to the basic graph of $y = x^2$?

What happens when k is a positive number?

What happens when k is a negative number?

In the equation $y = x^2 + k$, where is the vertex?

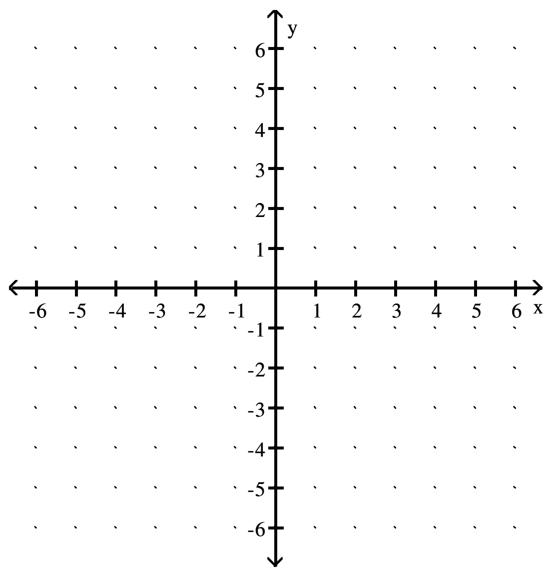
Graphing $y = (x - h)^2$

4. Delete all of your equations except for $y = x^2$ in equation 1.
   In equation 2, graph $y = (x - 2)^2$. Click on the graph button and you will see your original $y = x^2$ in red and your new graph $y = (x - 2)^2$ on the same axes graphed in blue. Be sure your parenthesis are correct! You will type (x – 2)^2 here.

   What happened? Did replacing x with x – 2 cause the vertex to move? Where is the vertex of the graph $y = (x - 2)^2$?

5. Click on the equation button and in equation 3, type $y = (x + 4)^2$. What is the vertex of the graph $y = (x + 4)^2$?

6. Sketch both equations, $y = (x - 2)^2$ and $y = (x + 4)^2$ on the plane below. Label which graph is which and label the vertex on each graph.



Try graphing $y = (x - h)^2$ with various values for h until you are confident that you know what the graph will look like.

<u>Generalize the concept:</u>

How does the graph of $y = (x - h)^2$ compare to the basic graph of $y = x^2$? Consider what happens when h is positive and when h is negative.

In the equation $y = (x - h)^2$, where is the vertex? Do you understand why mathematicians put the minus sign in this form?

Graphing $y = ax^2$

7. Delete all of your equations except for $y = x^2$ in equation 1.
   In equation 2, graph $y = 3x^2$. You need to type 3*x^2. Click on the graph button and you will see your original $y = x^2$ in red and your new graph $y = 3x^2$ on the same axes graphed in blue.

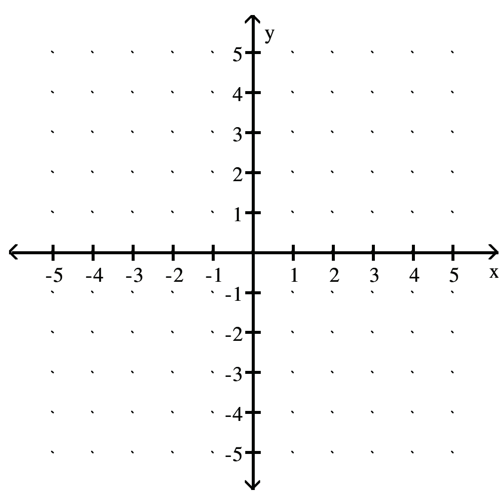
   What happened? Did multiplying by 3 cause the vertex to move to a different location? Did multiplying by 3 change the shape of the parabola?

8. Click on the equation button and in equation 3, type $y = 5*x^2$. How does this graph compare with the previous two?

9. Click on the equation button and in equation 4, type $y = \left(\frac{1}{3}\right) * x^2$
   and in equation 5 type $y = \left(\frac{1}{8}\right) * x^2$. Click on the graph button and view these graphs.

What do you see?

10. Sketch all of the equations, $y = 2x^2$, $y = 5x^2$, $y = \left(\frac{1}{3}\right)x^2$, and $y = \left(\frac{1}{8}\right)x^2$ on the plane below. Label which graph is which.



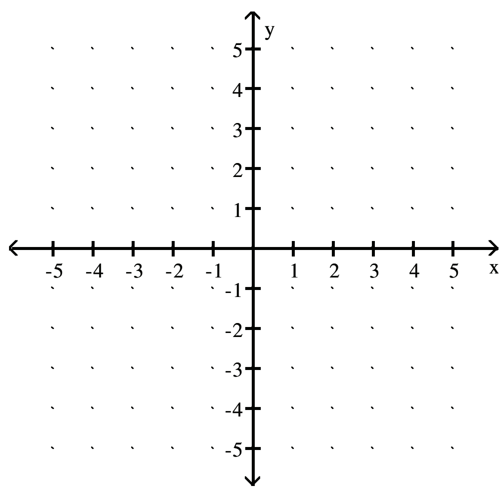
11. What happens if a is a negative number? Delete all of your equations except for $y = x^2$ in equation 1.

In equation 2, graph $y = -x^2$. Click on the graph button and you will see your original $y = x^2$ in red and your new graph $y = -x^2$ on the same axes graphed in blue.

What happened?

12. Click on the equation button and in equation 3, type $y = -4*x^2$ and in equation 4, type $y = \left(-\frac{1}{5}\right) * x^2$. Did your graphs look the way you expected them to look?

13. Sketch all of the equations, $y = -x^2$, $y = -4x^2$, and $y = \left(-\frac{1}{5}\right)x^2$ on the plane below. Label which graph is which.



Try graphing $y = ax^2$ with various values for a until you are confident that you know what the graph will look like.

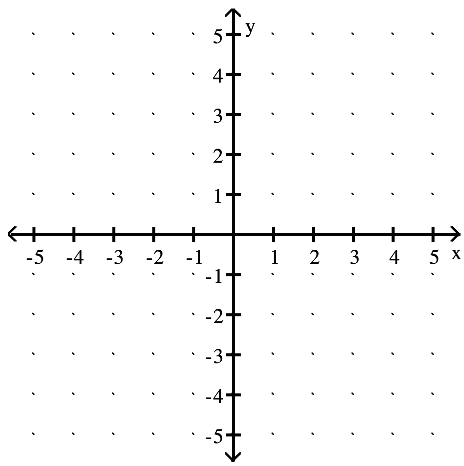
Generalize the concept:

How does the graph of $y = ax^2$ compare to the basic graph of $y = x^2$? Consider what happens when a is positive and when h is negative. What happens when $|a| > 1$? What happens when $0 < |a| < 1$?

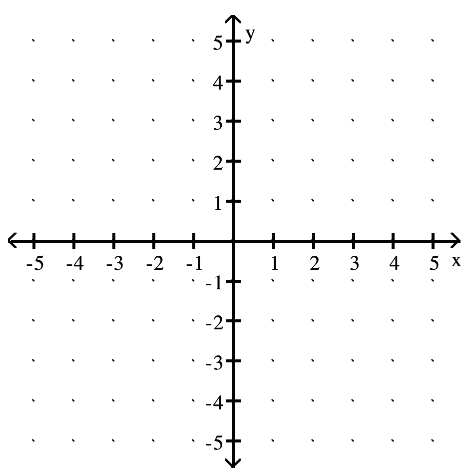
PART IV: Putting it All Together, the graph of $f(x) = a(x - h)^2 + k$

Graph and sketch each of the following functions. State the vertex and the equation of the axis of symmetry. Based on what you have been learning, try to predict what the graph will look like before you click on the graph button.

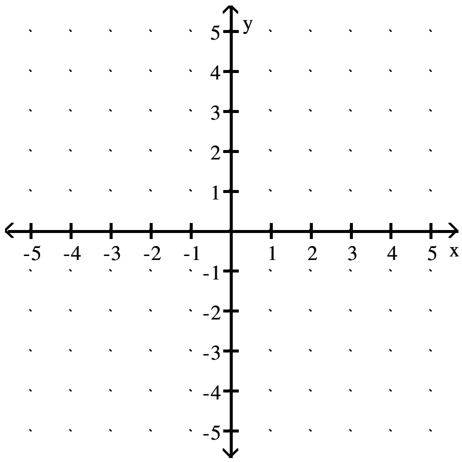
1. $f(x) = (x + 3)^2 - 4$ vertex ______________ axis of symmetry ___________



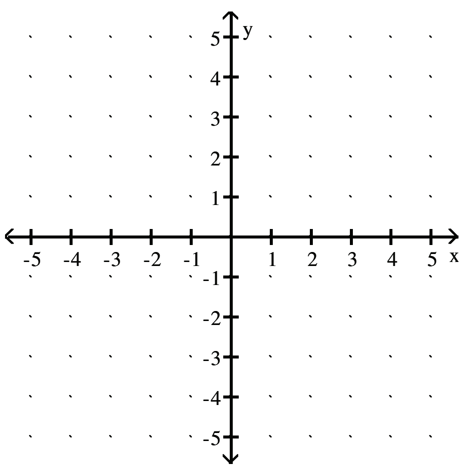
2. $g(x) = -2(x - 1)^2 + 5$ vertex ______________ axis of symmetry ___________



3. $f(x) = 0.2\,(x + 1)^2 + 2$ vertex ______________ axis of symmetry __________



4. $f(x) = 3(x - 4)^2$ vertex ______________ axis of symmetry __________



Experiment by graphing $f(x) = a(x - h)^2 + k$ using different values of a, h, and k until you can consistently predict what the graph will look like before you click the graph button.

In general:
The vertex is located at the point ____________________

The equation of the axis of symmetry is ___________________________

If a is positive the graph will open ______________

If a is negative the graph will open ______________

If $|a| > 1$ the graph will appear __________________ than the graph of $y = x^2$

If $0 < |a| < 1$ the graph will appear __________________ than the graph of $y = x^2$