Name________________________________

Activity - Derivatives of Trig Functions

Part 1.

Consider the function $f(x) = \sin(x)$, which is graphed in the figure below. Note carefully that the grid in the diagram does not have boxes that are 1×1, but rather approximately 1.57×1, as the horizontal scale of the grid is $\pi/2$ units per box.

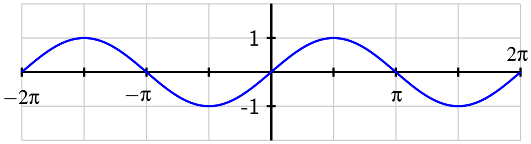
(a) At each of $x = -2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$, use a straightedge to sketch an accurate tangent line to $y = f(x)$.

(b) Use the provided grid to estimate the slope of the tangent line you drew at each point. Pay careful attention to the scale of the grid.

(c) Use the average rate of change formula to estimate $f'(0)$ by using small values of h. Make a table with h = -0.1, 0.1, -0,01, 0.01, -0.001 and 0.001 and compare the result to your visual estimate for the slope of the tangent line to $y = f(x)$ at $x = 0$ in (b). Using periodicity, what does this result suggest about $f'(2\pi)$? about $f'(-2\pi)$?

(d) Based on your work in (a), (b), and (c), sketch an accurate graph of $y = f'(x)$ on the axes adjacent to the graph of $y = f(x)$.

(e) What familiar function do you think is the derivative of $f(x) = \sin(x)$?



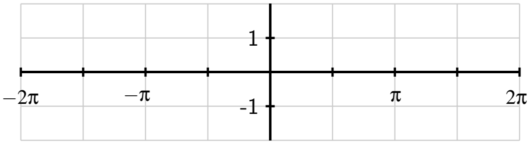


Figure: At left, the graph of $y = f(x) = \sin(x)$.

Part 2.

Consider the function $g(x) = \cos(x)$, which is graphed in the figure below. Note carefully that the grid in the diagram does not have boxes that are 1×1, but rather approximately 1.57×1, as the horizontal scale of the grid is $\pi/2$ units per box.

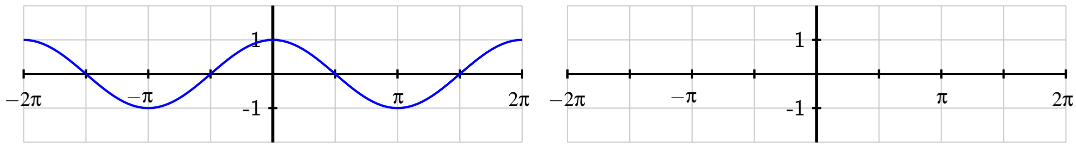


Figure: At left, the graph of $y = g(x) = \cos(x)$.

(a) At each of $x = -2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$, use a straightedge to sketch an accurate tangent line to $y = g(x)$.

(b) Use the provided grid to estimate the slope of the tangent line you drew at each point. Again, note the scale of the axes and grid.

(c)Use the average rate of change formula to estimate $g'\left(\frac{\pi}{2}\right)$ by using small values of h, and compare the result to your visual estimate for the slope of the tangent line to $y = g(x)$ at $x \frac{\pi}{2}$ in (b). Using periodicity, what does this result suggest about $g'\left(-\frac{3\pi}{2}\right)$?

(d) Based on your work in (a), (b), and (c), sketch an accurate graph of $y = g'(x)$ on the axes adjacent to the graph of $y = g(x)$.

(e) What familiar function do you think is the derivative of $g(x) = \cos(x)$?

Part 3. Consider the function $f(x) = \tan(x)$, and remember that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$.

(a) What is the domain of f?

(b) Use the quotient rule to show that one expression for $f'(x)$ is

$$f'(x) = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos^2(x)}.$$

(c) Use Pythagorean identities from trigonometry to find a simpler form for $f'(x)$.

(d) Recall that $\sec(x) = \frac{1}{\cos(x)}$. How can we express $f'(x)$ in terms of the secant function?

(e) For what values of x is $f'(x)$ defined? How does this set compare to the domain of f?

Part 4.

Let $h(x) = \sec(x)$ and recall that $\sec(x) = \frac{1}{\cos(x)}$.

(a) What is the domain of h?

(b) Use the quotient rule to develop a formula for $h'(x)$ that is expressed completely in terms of $\sin(x)$ and $\cos(x)$.

(c) How can you use other relationships among trigonometric functions to write $h'(x)$ only in terms of $\tan(x)$ and $\sec(x)$?

(d) What is the domain of h'? How does this compare to the domain of h?

Part 5.

Let $p(x) = \csc(x)$ and recall that $\csc(x) = \frac{1}{\sin(x)}$.

(a) What is the domain of p?

(b) Use the quotient rule to develop a formula for $p'(x)$ that is expressed completely in terms of $\sin(x)$ and $\cos(x)$.

(c) How can you use other relationships among trigonometric functions to write $p'(x)$ only in terms of $\cot(x)$ and $\csc(x)$?

(d) What is the domain of p'? How does this compare to the domain of p?

Part 6.

Using the same techniques as in the previous parts, find the derivative of $k(x) = \cot(x)$. What is the domain of $k(x)$?