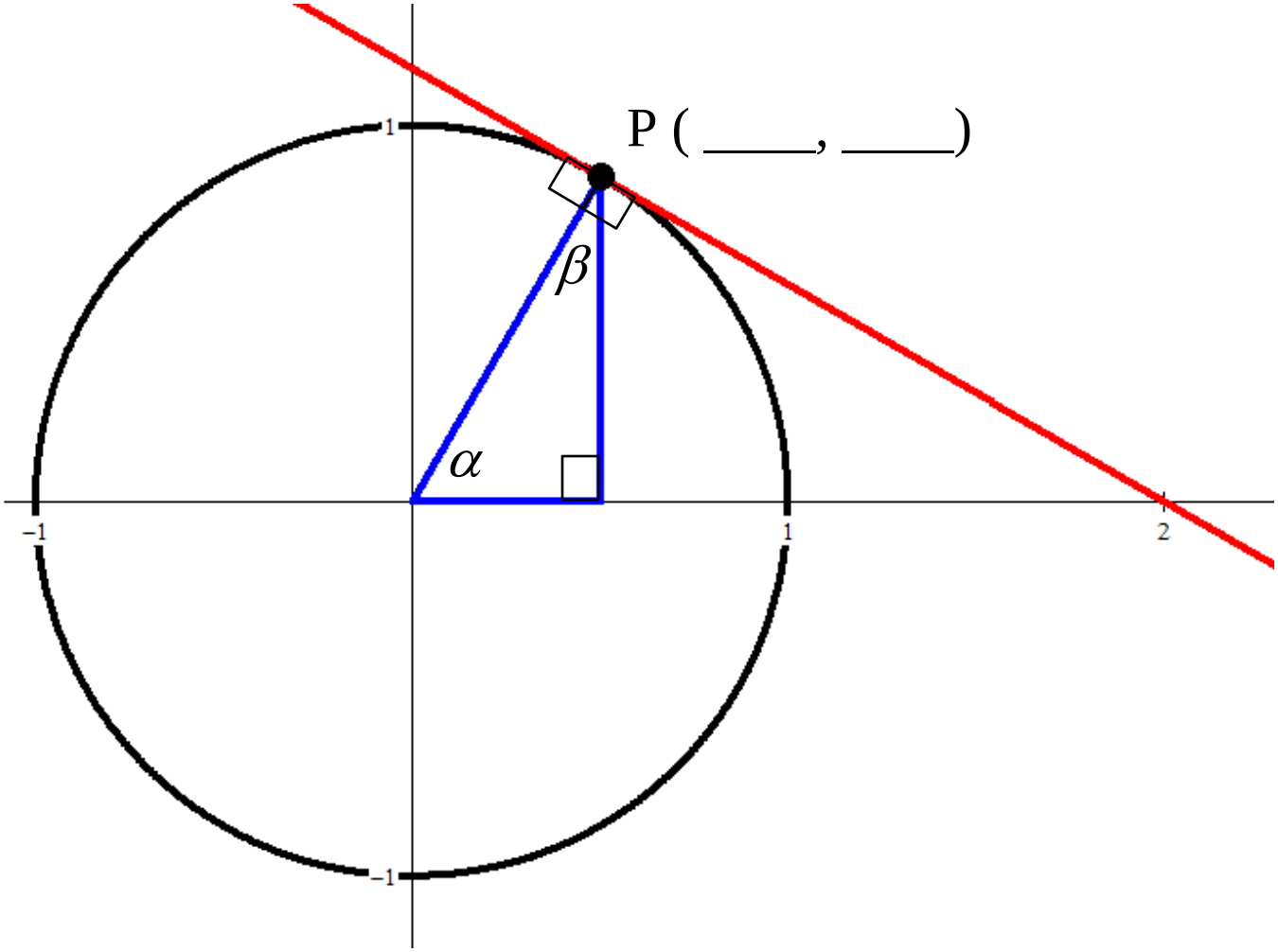
Name_____

Activity – Discovering the Pythagorean Identities

Part 1. The figure below is the unit circle with point P and a line tangent to the circle at point P.
Use α as the angle in all trig functions on this activity.

1. Fill in point P with the correct trig functions and label the lengths of the 3 sides of the triangle with constants or trig functions. Use the Pythagorean Theorem to come up with an identity that connects these trig functions.



2. a) Notice that the hypotenuse of the original triangle and a segment on the tangent line form legs of a right triangle whose hypotenuse is a piece of the y-axis. Trace this triangle and label its angles. Hint: There will be a right angle, and angles α and β.

b) Use similar triangles to find the sides of the new triangle. Use quotient and reciprocal identities to express these sides without fractions.

c) Now use the Pythagorean Theorem to come up with an identity that connects these two trig functions.

3. Notice that the hypotenuse of the original triangle and a segment on the tangent line form legs of a right triangle whose hypotenuse is a piece of the *x*-axis. Trace this triangle and label its angles again using α and β and then repeat steps b) and c) in 2 above to find another identity that connects these trig functions.

Part 2. Another way to derive the Pythagorean Identities is to start with the identity found in a) of part 1. This identity is often also called the Fundamental Identity because of its fundamental and frequent use in trig.

a) Write the fundamental identity.

b) Divide all terms in the fundamental identity by $\cos^2\alpha$.

c) Use reciprocal identities to express this identity without fractions. Does this identity match an identity derived on page 1? If so which part?

d) Write the fundamental identity again.

e) This time, divide all terms in the fundamental identity by $\sin^2\alpha$.

f) Use reciprocal identities to express this identity without fractions. Does this identity match an identity derived on page 1? If so which part?

Part 3. Complete the table below. Alternate forms should solve for a different term (no need to square root though), so for example in the first one, an alternate form would be $\cos^2\alpha =$ _____ .

	Original Form	Alternate Form 1	Alternate Form 2
Pythagorean Identity Connecting $\sin\alpha$ and $\cos\alpha$			
Pythagorean Identity Connecting $\cot\alpha$ and $\csc\alpha$			
Pythagorean Identity Connecting $\tan\alpha$ and $\sec\alpha$			