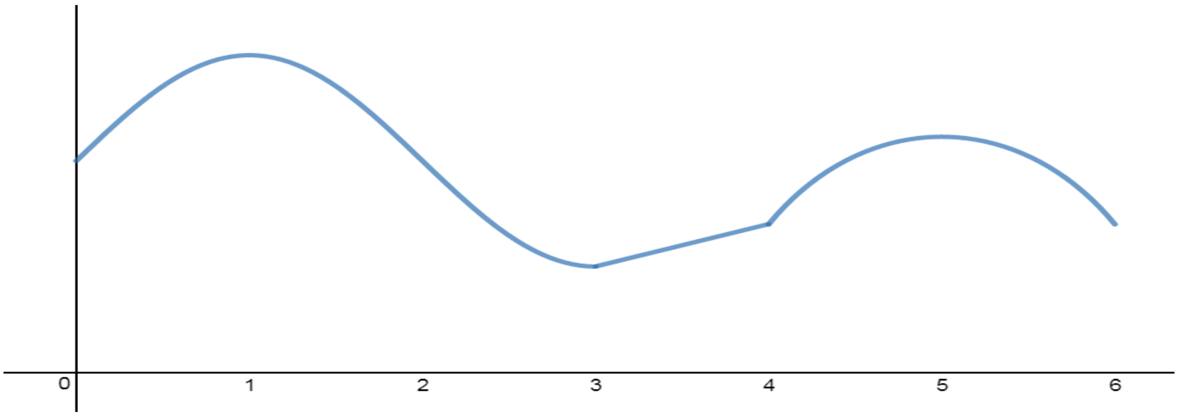
Name___________________________________

WhatDerivatives Tell Us About Graphs

Part 1



Looking at the graph above answer the following:

a) On what intervals is the function increasing?

b) On these intervals, what is the sign of $f'(x)$?

c) On what intervals is the function decreasing?

d) On these intervals, what is the sign of $f'(x)$?

e) On what intervals is the function constant?

f) On these intervals, what is the sign of $f'(x)$?

g) For what *x* values does the function have a local maximum?

h) What is the sign of the derivative on either side of the local maximum?

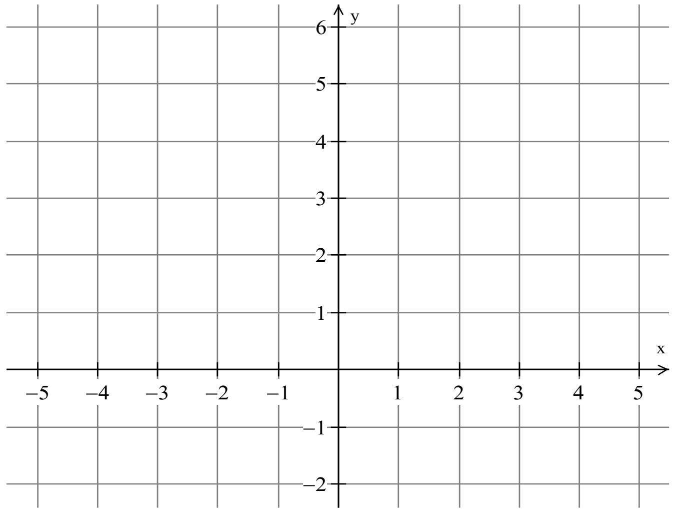
i) For what *x* values does the function have a local minimum?

j) What is the sign of the derivative on either side of the local minimum?

Part 2 What Derivatives Tell Us About the Graph of a Function

1. Using a graphing program such as desmos.com, graph the function $f(x) = \ln(3x^2+1) - 1$ and sketch the graph.

Graph of $f(x)$



From this graph, predict some of the key properties of the graphs of its first and second derivatives:

a) On what interval(s) is $f(x)$ increasing?
What must be true about the graph of $f'(x)$ here?

b) On what interval(s) is $f(x)$ decreasing? What must be true about the graph of $f'(x)$ here?

c) At what x value(s) does $f(x)$ have local maxima or minima?
What must be true about the graph of $f'(x)$ at these points?

d) Will the graph of $f'(x)$ have any local maximum points? If so, where?

e) Will the graph of $f'(x)$ have any local minimum points? If so, where?

f) On approximately what interval(s) is $f(x)$ concave up?
What must be true about the graph of $f''(x)$ here?

g) On approximately what interval(s) is $f(x)$ concave down?
What must be true about the graph of $f''(x)$ here?

h) At what x value(s) does $f(x)$ have inflection points?
What must be true about the graph of $f''(x)$ at these points?

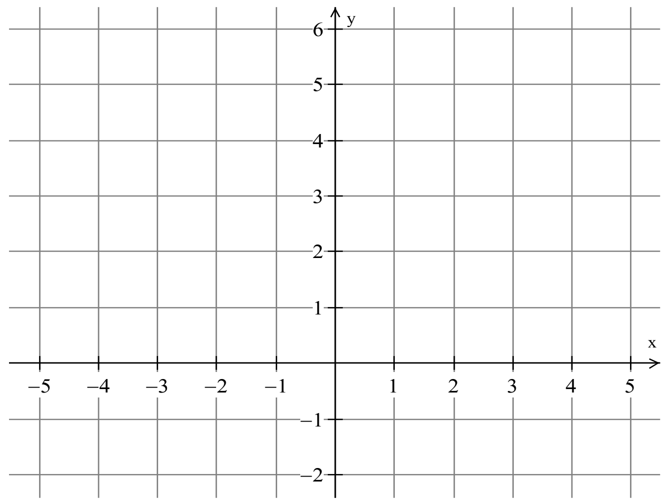
2. Now calculate the first derivative of $f(x) = \ln(3x^2 + 1) - 1$

$f'(x) =$ ______________________

Use the graphing program to graph this function and sketch the graph below.

Does the graph of $f'(x)$ agree with your predictions above? (If not, find your errors and correct them.)

Graph of $f'(x)$



Now consider this new graph:

a) On what interval(s) is $f'(x)$ increasing?

What must be true about the graph of $f''(x)$ here?

b) On what interval(s) is $f'(x)$ decreasing?

What must be true about the graph of $f''(x)$ here?

c) At what *x* value(s) does $f'(x)$ have local maxima or minima?

What must be true about the graph of $f''(x)$ at these points?

d) Will the graph of $f''(x)$ have any local maximum points? If so, where?

e) Will the graph of $f''(x)$ have any local minimum points? If so, where?

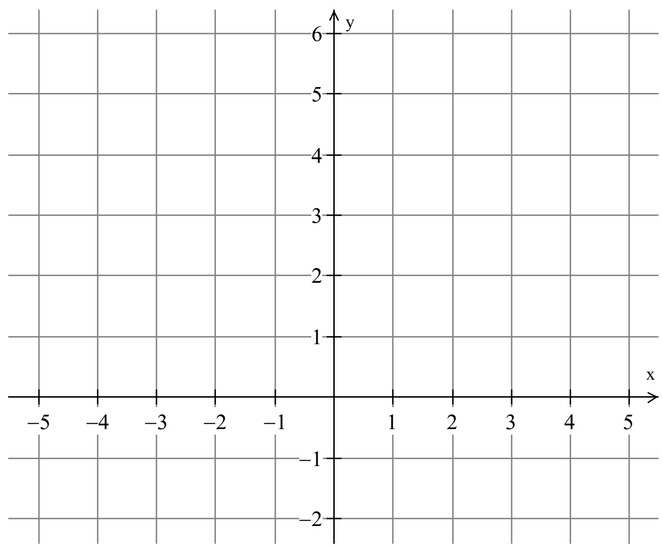
3. Finally, calculate the second derivative of $f(x) = \ln(3x^2 + 1) - 1$,

$f''(x) =$ ________________________

Use the graphing program to graph this function and sketch the graph below.

Does the graph of $f''(x)$ agree with your predictions above? (If not, find your errors and correct them.)

Graph of $f''(x)$

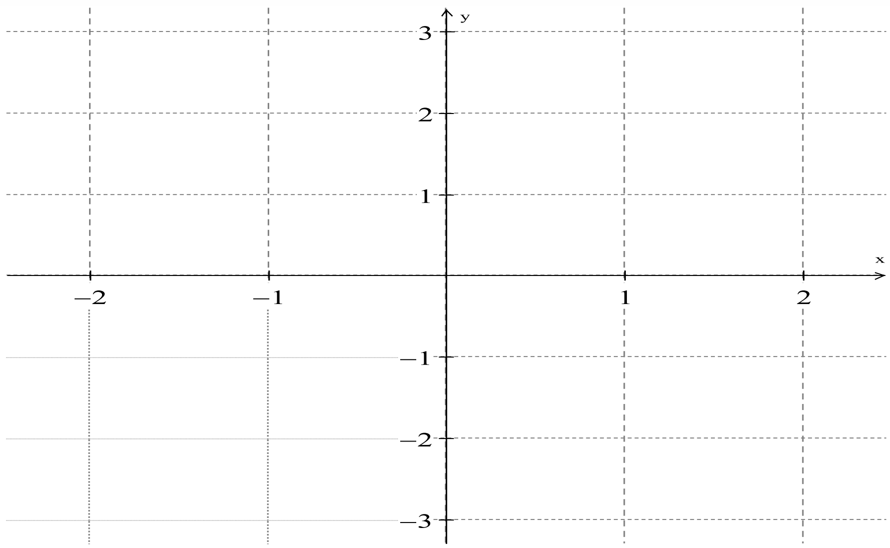


Use your graphing program, graph all three functions together and study your graphs carefully to see the relationships between them. Write at least three sentences explaining those relationships.

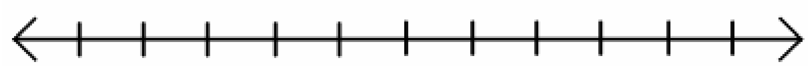
Part 3.

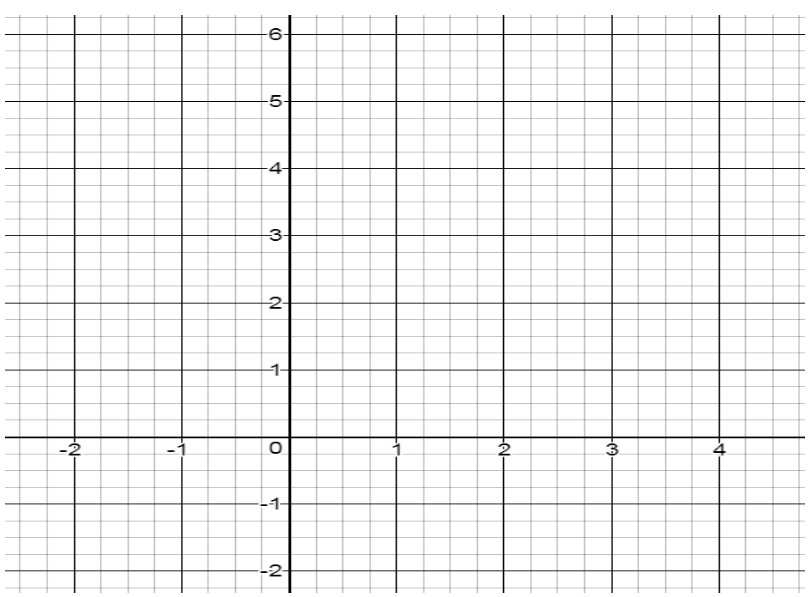
a) Graph a continuous function f(x) which satisfies the conditions for f and f ′ given below:

x	–2	–1	0	1	2
f(x)	1	–1	–2	–1	0
f '(x)	–1	0	1	2	–1



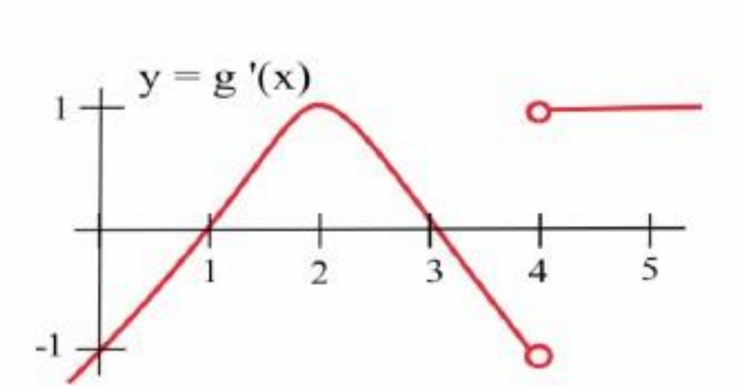
b) Construct a number line with signs of the first derivative of $f(x)=x^3-6x^2+9x+1$ and use this information to graph *f*(*x*).



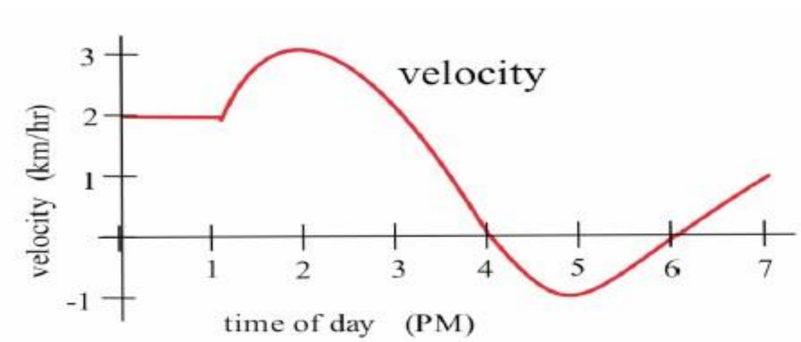


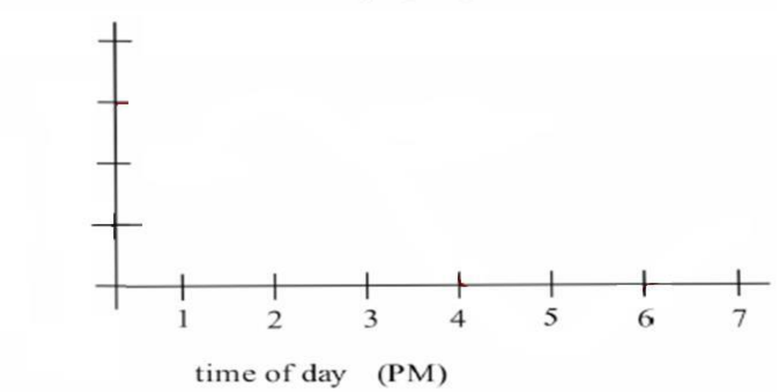
Part 4.

a) Use the graph of $g'(x)$ to sketch the shape of the graph of $g(x)$.



b) A weather balloon is released from the ground and sends back its velocity measurements given as a graph. Sketch a graph of the height of the balloon based on these measurements. At what tine was the balloon the highest?

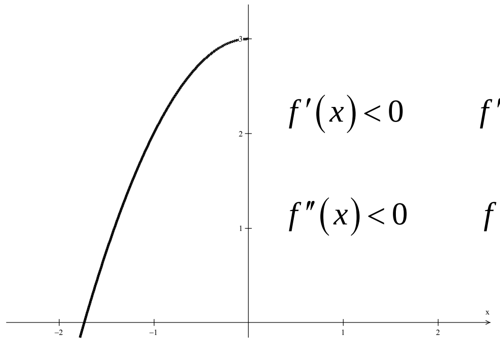




Part 5

Circle the correct statements about the derivatives for each graph.

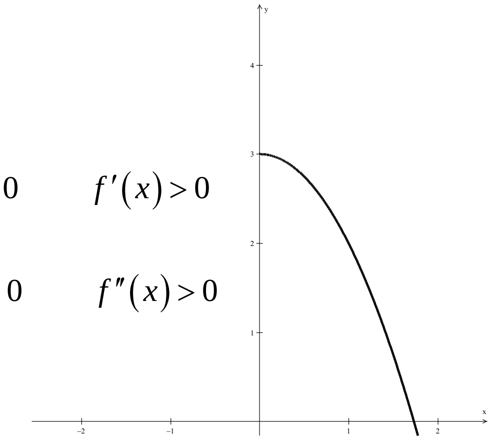
a)



$f'(x) < 0$ $f'(x) > 0$

$f''(x) < 0$ $f''(x) > 0$

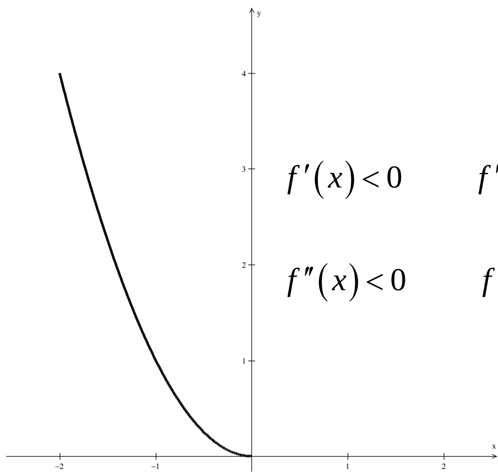
b)



$f'(x) < 0$ $f'(x) > 0$

$f''(x) < 0$ $f''(x) > 0$

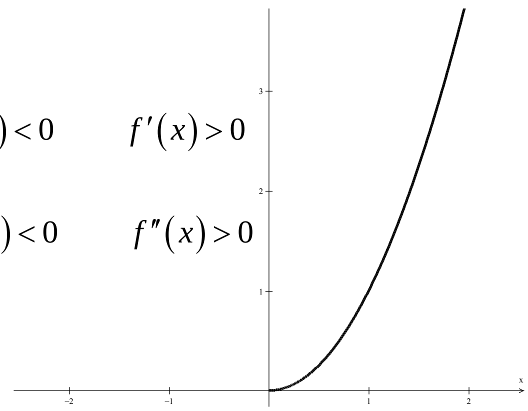
c)



$f'(x) < 0$ $f'(x) > 0$

$f''(x) < 0$ $f''(x) > 0$

d)



$f'(x) < 0$ $f'(x) > 0$

$f''(x) < 0$ $f''(x) > 0$